# Reteach

# 11-3 Multiplying and Dividing Radical Expressions

Use the Product and Quotient Properties to multiply and divide radical expressions.

Product Property of Square Roots	Quotient Property of Square Roots	
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ ; where $a \ge 0$ and $b \ge 0$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ; where $a \ge 0$ and $b > 0$	

### Multiply $\sqrt{6} \sqrt{10}$ .

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 $\sqrt{6 \cdot 10}$  Product Property of Square

Roots

 $\sqrt{60}$  Multiply the factors in the

radicand.

 $\sqrt{4 \cdot 15}$  Factor 60 using a perfect

square factor.

 $\sqrt{4} \cdot \sqrt{15}$  Product Property of Square

Roots

 $2\sqrt{15}$  Simplify.

A quotient with a square root in the denominator is not simplified. Rationalize the denominator by multiplying by a form of 1 to get a perfect square.

# Simplify $\sqrt{\frac{10}{3}}$ .

$$\sqrt{\frac{10}{3}} = \frac{\sqrt{10}}{\sqrt{3}}$$

 $\frac{\sqrt{10}}{\sqrt{3}} \left( \frac{\sqrt{3}}{\sqrt{3}} \right)$  Multiply by form of 1.

Product Property

$$\frac{\sqrt{30}}{3}$$

Simplify.

**Quotient Property** 

### Multiply. Then simplify.

**1.** 
$$\sqrt{3} \sqrt{12}$$

**2.** 
$$\sqrt{5} \sqrt{10}$$

**3.** 
$$\sqrt{8} \sqrt{11}$$

### Rationalize the denominator of each quotient. Then simplify.

4. 
$$\frac{\sqrt{7}}{\sqrt{2}} \left( \frac{\phantom{0}}{\phantom{0}} \right)$$

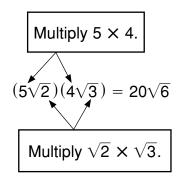
$$5. \ \frac{\sqrt{8}}{\sqrt{3}} \left( \frac{\boxed{\phantom{0}}}{\boxed{\phantom{0}}} \right)$$

$$6. \frac{\sqrt{12}}{\sqrt{5}} \left( \frac{\phantom{0}}{\phantom{0}} \right)$$

## **LESSON** Reteach

# 11-8 Multiplying and Dividing Radical Expressions (continued)

Terms can be multiplied and divided if they are both under the radicals OR if they are both outside the radicals.



Multiply  $\sqrt{3}$  (6 +  $\sqrt{8}$ ). Write the product in simplest form.

$$\sqrt{3} (6 + \sqrt{8})$$

$$\sqrt{3}$$
(6) +  $\sqrt{3}$   $\sqrt{8}$  Distribute.

$$6\sqrt{3} + \sqrt{24}$$
 Multiply the factors in

$$6\sqrt{3} + \sqrt{4 \cdot 6}$$
 Factor 24 using a perfect square factor.

$$6\sqrt{3} + \sqrt{4}\sqrt{6}$$
 Product Property of Square Roots

$$6\sqrt{3} + 2\sqrt{6}$$
 Simplify.

Use FOIL to multiply binomials with square roots.

Multiply  $(3 + \sqrt{2}) (4 + \sqrt{2})$ .

$$(3+\sqrt{2})(4+\sqrt{2})$$

$$3(4) + 3\sqrt{2} + 4\sqrt{2} + \sqrt{2}\sqrt{2}$$
 FOIL.

$$12 + 3\sqrt{2} + 4\sqrt{2} + \sqrt{4}$$
 Multiply.

$$12 + 3\sqrt{2} + 4\sqrt{2} + 2$$
 Simplify.

$$14 + 7\sqrt{2}$$
 Add.

Multiply. Write each product in simplest form.

**7.** 
$$\sqrt{5} (4 + \sqrt{8})$$

$$\sqrt{5}$$
 +  $\sqrt{5}$ 

**8.** 
$$\sqrt{2} (\sqrt{2} + \sqrt{14})$$

**9.**  $(6 + \sqrt{3})(5 - \sqrt{3})$ 

$$(6) \Big( \Big) - (6) \Big( \Big) + \sqrt{3} \Big( \Big) - \sqrt{3} \Big( \Big) \Big]$$

**10.** 
$$(5 + \sqrt{10})(8 + \sqrt{10})$$

#### TESSON Practice A 1158 Multiplying and Dividing Radical Expressions Multiply. Write each product in simplest form. 1. $\sqrt{3} \cdot \sqrt{15}$ **2.** $(2\sqrt{7})^2$ **3.** $3\sqrt{5t} \cdot \sqrt{40t}$ $\sqrt{3\cdot 15}$ $2\sqrt{7}\cdot 2\sqrt{7}$ $3 \cdot \sqrt{(5t)(40t)}$ $\sqrt{45}$ $\mathbf{2} \, \cdot \, \mathbf{2} \cdot \sqrt{7} \cdot \sqrt{7}$ $3 \cdot \sqrt{200t^2}$ $\sqrt{9\cdot 5}$ ${4\hspace{-.1em}\rule{0.8em}{0.8em}\hspace{0.1em}\rule{0.8em}{0.8em}\hspace{0.1em}} 4 \cdot \sqrt{7 \cdot 7}$ $3 \cdot \sqrt{2 \cdot 100 \cdot t^2}$ $4 \cdot \sqrt{49}$ $3\sqrt{5} \\$ $30t\sqrt{2}$ **4.** $\sqrt{10} \cdot \sqrt{5}$ 5. $(3\sqrt{10})^2$ $12x\sqrt{14}$ $5\sqrt{2}$ 7. $\sqrt{3}(\sqrt{6}-2)$ **8.** $\sqrt{6}(\sqrt{2} - \sqrt{3t})$ **9.** $(2-\sqrt{5})(7+\sqrt{5})$ $\sqrt{6}(\sqrt{2}) - \sqrt{6}(\sqrt{3t})$ $\sqrt{3}(\sqrt{6}) - \sqrt{3}(2)$ $14 + 2\sqrt{5} - 7\sqrt{5} - 5$ $\sqrt{12} - \sqrt{18t}$ $\sqrt{18}$ – $2\sqrt{3}$ $3\sqrt{2}-2\sqrt{3}$ $2\sqrt{3}-3\sqrt{2t}$ $9-5\sqrt{5}$ **11.** $\sqrt{7}(\sqrt{7} + \sqrt{5})$ **12.** $(3 + \sqrt{2})(\sqrt{2} - 4)$ **10.** $\sqrt{5}(\sqrt{5} - 8)$ $\mathbf{5-8}\sqrt{\mathbf{5}}$ $7 + \sqrt{35}$ $-\sqrt{2}-10$ Simplify each quotient. 13. $\frac{\sqrt{3}}{\sqrt{5}}$ √10*b* 8*b* 17. $\frac{\sqrt{10}}{\sqrt{2}}$ **18.** $(4 + \sqrt{3})(5 - \sqrt{3})$ $\sqrt{30}$ $\sqrt{5}$ $17 + \sqrt{3}$ 59 Holt Algebra Copyright © by Holt, Rinehart and Wins All rights reserved. **Practice C** 1. $\sqrt{15} \cdot \sqrt{5}$ **2.** $\sqrt{42} \cdot \sqrt{12}$ 3. $(2\sqrt{10})^2$ $5\sqrt{3}$ $6\sqrt{14}$ 40

<b>1.</b> $\sqrt{15} \cdot \sqrt{6}$	<b>2.</b> $(3\sqrt{6})^2$	<b>3.</b> $4\sqrt{7x} \cdot \sqrt{20x}$
$\sqrt{15\cdot 6}$	$3\sqrt{6} \cdot 3\sqrt{6}$	$4\cdot\sqrt{(7x)(20x)}$
3√10	54	$8x\sqrt{35}$
<b>4.</b> $\sqrt{12} \cdot \sqrt{5}$	5. $(2\sqrt{7})^2$	<b>6.</b> $-2\sqrt{5b} \cdot \sqrt{10b}$
<u>2√15</u>	28	$-10b\sqrt{2}$
<b>7.</b> $3\sqrt{10y}\sqrt{6y}$	<b>8.</b> $\sqrt{8}(\sqrt{12} - \sqrt{2})$	$9. \sqrt{2x}(\sqrt{5} + \sqrt{2x})$
$6y\sqrt{15}$	$\frac{4\sqrt{6}-4}{\sqrt{6}}$	$\sqrt{10x} + 2x$
<b>10.</b> $\sqrt{2}(\sqrt{7}-5)$	<b>11.</b> $\sqrt{10}(\sqrt{5m} - \sqrt{4})$	<b>12.</b> $(4 + \sqrt{3})(2 - \sqrt{3})$
$\frac{\sqrt{14} - 5\sqrt{2}}{\sqrt{3}(\sqrt{8} - 6)}$	$-\frac{5\sqrt{2m}-2\sqrt{10}}{14.\sqrt{5}(\sqrt{2}+\sqrt{8})}$	
$\frac{2\sqrt{6} - 6\sqrt{3}}{\sqrt{5}(\sqrt{2} - \sqrt{6})}$	$ \frac{3\sqrt{10}}{17. (3-\sqrt{2})(5+\sqrt{2})} $	18. $(7 + \sqrt{3})(7 - \sqrt{3})$
$\sqrt{10} - \sqrt{30}$	$13-2\sqrt{2}$	46
Simplify each quotient.		
<b>19.</b> $\frac{\sqrt{2}}{\sqrt{6}}$	<b>20.</b> $\frac{\sqrt{10}}{\sqrt{11}}$	<b>21.</b> $\frac{\sqrt{13}}{\sqrt{50t}}$
$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{110}}{11}$	$\frac{\sqrt{26t}}{10t}$
<b>22.</b> $\frac{\sqrt{7}}{\sqrt{15}}$	<b>23.</b> $\frac{\sqrt{2}}{\sqrt{17}}$	<b>24.</b> $\frac{\sqrt{32}}{\sqrt{48z}}$
$\frac{\sqrt{105}}{15}$	$\frac{\sqrt{34}}{17}$	$\frac{\sqrt{6z}}{3z}$
25. $\frac{\sqrt{3}}{\sqrt{3}a}$	<b>26.</b> $\frac{\sqrt{8x}}{\sqrt{5}}$	27. $-\frac{\sqrt{75k}}{10\sqrt{2k}}$
$\frac{\sqrt{a}}{a}$	$\frac{2\sqrt{10x}}{5}$	$-\frac{\sqrt{6}}{4}$

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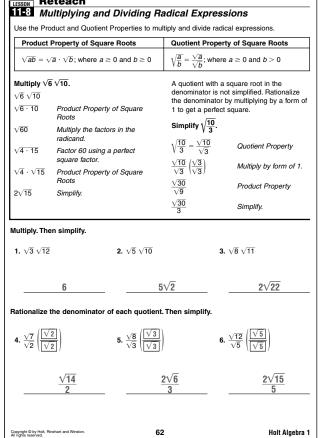
#### 11-3 Multiplying and Dividing Radical Expressions Multiply. Write each product in simplest form. 4. $(5\sqrt{5})^2$ **5.** $3\sqrt{6x} \cdot \sqrt{10x}$ **6.** $4\sqrt{6x} \cdot \sqrt{12x}$ 125 $6x\sqrt{15}$ $24x\sqrt{2}$ 7. $\sqrt{3}(\sqrt{12}+6)$ **8.** $\sqrt{6}(\sqrt{10c} - \sqrt{8})$ **9.** $(10 + \sqrt{5})(4 - \sqrt{5})$ $2\sqrt{15c} - 4\sqrt{3}$ $6 + 6\sqrt{3}$ $35 - 6\sqrt{5}$ **10.** $\sqrt{7}(\sqrt{14} + 2)$ **11.** $\sqrt{3}(\sqrt{3} - \sqrt{6})$ **12.** $(9 - \sqrt{3})(4 - \sqrt{3})$ $7\sqrt{2} + 2\sqrt{7}$ $3 - 3\sqrt{2}$ $39 - 13\sqrt{3}$ **14.** $(2\sqrt{5} - \sqrt{3})(\sqrt{5} - \sqrt{3})$ **15.** $(9 - \sqrt{3})^2$ **13.** $(4 + \sqrt{5})(1 - \sqrt{5})$ $13 - 3\sqrt{15}$ $-1 - 3\sqrt{5}$ $84 - 18\sqrt{3}$ Simplify each quotient. 16. $\frac{\sqrt{3}}{\sqrt{5}}$ 17. $\frac{\sqrt{8}}{\sqrt{3}}$ $\sqrt{15}$ $2\sqrt{6}$ 19. $\frac{\sqrt{18}}{\sqrt{2}}$ $\sqrt{6x}$

 $2\sqrt{3x}$ 

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25. Find the area of a triangle whose base is given by the expression  $3\sqrt{6}$  m and whose height is given by the expression  $2\sqrt{8}$  m.

**26.** The area of <u>a</u> rectangle is  $3\sqrt{50}$  yd<sup>2</sup>. Find the width if the



3

 $\sqrt{22x}$ 

length is 3√5 yd

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12√3 m<sup>2</sup>

 $\sqrt{10}$  yd

**22.**  $\frac{\sqrt{11}}{\sqrt{72x}}$ 

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**24.**  $-\frac{\sqrt{200m}}{2\sqrt{3m}}$ 

 $5\sqrt{6}$ 

# Reteach 1153 Multiplying and Dividing Radical Expressions (continued)

Terms can be multiplied and divided if they are both under the radicals OR if they are both



#### Multiply $\sqrt{3}$ (6 + $\sqrt{8}$ ). Write the product in simplest form.

 $\sqrt{3} (6 + \sqrt{8})$ 

 $\sqrt{3}(6) + \sqrt{3} \sqrt{8}$ Distribute.

 $6\sqrt{3} + \sqrt{24}$ Multiply the factors in the radicand.  $6\sqrt{3} + \sqrt{4\cdot 6}$ Factor 24 using a perfect square factor.

 $6\sqrt{3} + \sqrt{4}\sqrt{6}$ Product Property of Simplify.

 $6\sqrt{3} + 2\sqrt{6}$ 

Use FOIL to multiply binomials with square roots.

Multiply  $(3 + \sqrt{2}) (4 + \sqrt{2})$ .

 $(3 + \sqrt{2})(4 + \sqrt{2})$ 

 $3(4) + 3\sqrt{2} + 4\sqrt{2} + \sqrt{2}\sqrt{2}$ FOIL.  $12 + 3\sqrt{2} + 4\sqrt{2} + \sqrt{4}$ Multiply  $12 + 3\sqrt{2} + 4\sqrt{2} + 2$ Simplify

 $14 + 7\sqrt{2}$ Add.

#### Multiply. Write each product in simplest form.

**7.** 
$$\sqrt{5} (4 + \sqrt{8})$$

**8.** 
$$\sqrt{2} (\sqrt{2} + \sqrt{14})$$

$$\sqrt{5}$$
 4 +  $\sqrt{5}$   $\sqrt{8}$ 

$$4\sqrt{5} + 2\sqrt{10}$$

$$2 + 2\sqrt{7}$$

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**9.**  $(6 + \sqrt{3})(5 - \sqrt{3})$ 

$$(6)\Big(\boxed{5}\Big)-(6)\Big(\boxed{\sqrt{3}}\Big)+\sqrt{3}\Big(\boxed{5}\Big)-\sqrt{3}\Big(\boxed{\sqrt{3}}\Big)$$

$$27 - \sqrt{3}$$

**10.** 
$$(5 + \sqrt{10})(8 + \sqrt{10})$$

$$50 + 13\sqrt{10}$$

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### LESSON Challenge

### 11-8 Irrational Roots of Quadratic Equations

The solutions of a quadratic equation are sometimes called roots, but that has nothing to do with whether or not the solution contains a square root. If a solution does contain a square root, it is called an *irrational root*.

1. Is  $x = -5 + \sqrt{3}$  an irrational root of  $0 = x^2 + 10x + 22$ ? Substitute and simplify to find out.

Yes; 
$$(-5 + \sqrt{3})^2 + 10(-5 + \sqrt{3}) + 22 =$$
  
 $(28 - 10\sqrt{3}) + (-50 + 10\sqrt{3}) + 22 = 0$ 

**2.** Use the Quadratic Formula to find all roots of  $0 = x^2 + 10x + 22$ . Does this support your answer to guestion 1?

$$x=-5\pm\sqrt{3}$$
; yes,  $x=-5+\sqrt{3}$  is one of the two roots.

**3.** Is  $x = 1 - 3\sqrt{2}$  an irrational root of  $0 = 4x^2 - 4x - 17$ ? Substitute and simplify to find out.

No; 
$$4(1-3\sqrt{2})^2-4(1-3\sqrt{2})-17=$$

$$(76 - 24\sqrt{2}) - (4 - 12\sqrt{2}) - 17 = 55 - 12\sqrt{2} \neq 0$$

**4.** Use the Quadratic Formula to find all roots of  $0 = 4x^2 - 4x - 17$ . Does this support your answer to question 3?

$$x = \frac{1 \pm 3\sqrt{2}}{2} = \frac{1}{2} \pm \frac{3}{2}\sqrt{2}$$
; yes,  $x = 1 - 3\sqrt{2}$  is *not* one of the two roots.

- 5. Look at the roots that you found in questions 2 and 4. Based on these few examples, complete these conjectures about the irrational roots of quadratic equations that have rational coefficients:
  - a. In general, the irrational roots are \_\_\_\_ conjugates of each other.
- $m n\sqrt{p}$ **b.** If one root is  $m + n\sqrt{p}$ , then the other root is \_
- 6. Explain how the Quadratic Formula guarantees that your conjectures in question 5 will hold true any time a quadratic equation has irrational roots? The quadratic formula includes ± square root; irrational roots will always be conjugates.
- 7. Recall that when the discriminant  $(\it b^2-4\it ac)$  is greater than zero, there are two real solutions. What must be true about the discriminant for there to be two irrational solutions?

 $b^2 - 4ac$  would have to be greater than zero and *not* a perfect square.

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**Problem Solving** 

### 11-3 Multiplying and Dividing Radical Expressions

Write each correct answer as a radical expression in simplest form.

1. The expression  $\sqrt{\frac{W}{R}}$  models the electrical current in amperes, where *W* is power in watts and *R* is resistance in ohms. How much electrical current is running through an appliance with 500 watts of power and 16 ohms of resistance?

$$\frac{5\sqrt{5}}{2}$$
 amps

3. Riley's new bedroom is a perfect square. Find the area and perimeter of Rilev's

area: 12 m<sup>2</sup>
perimeter: 
$$8\sqrt{3}$$
 m

### Select the best answer.

- 4. R.J. lives in a studio apartment. The apartment is rectangular with a width of  $10 + 4\sqrt{2}$  feet and a length of + 11 $\sqrt{2}$  feet. What is the area of R.J.'s apartment?
  - A 60 ft<sup>2</sup>
- B 288 ft2
- $\mathbf{C} \ 200 + 190\sqrt{2} \ \text{ft}^2$
- $(\mathbf{D})$ 288 + 190 $\sqrt{2}$  ft<sup>2</sup>
- 6. The area of a rectangular window is 40 square feet. The length is  $\sqrt{20}$  feet. What is the width of the window?  $\mathbf{C}$   $4\sqrt{5}$  feet
- A  $\sqrt{2}$  feet
- $\mathbf{D}$   $4\sqrt{10}$  feet B 2 feet

2. The diagram shows the dimensions of a dining table. With a leaf in place, the table expands to seat eight people.



Find the area of the table.

$$448\sqrt{15} \text{ in}^2$$

Find the area of the table with the addition of a leaf that measures  $8\sqrt{6}$ inches by 18\sqrt{3} inches

$$448\sqrt{15} + 432\sqrt{2} \text{ in}^2$$

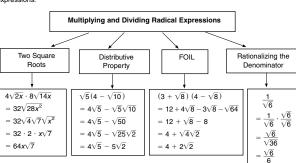
- 5. The volume of water in a lake, in gallons can be represented by  $x\sqrt{2}$ . Heavy rains are forecast. The volume of water is expected to increase  $\sqrt{2}$  times. How many gallons of water are expected in the lake after the rain?
  - $\mathbf{F} \stackrel{X}{=} \text{gallons}$
  - **G**  $\overset{\angle}{x}$  gallons
- H  $x\sqrt{2}$  gallons  $\bigcirc$  2x gallons
- 7. The height of a triangle can be found using  $h = \frac{2A}{b}$  where A is the area and b is the base of the triangle. Which shows the height of a triangle with an area of  $\sqrt{90}$  cm<sup>2</sup> and a base of  $\sqrt{5}$  cm written in simplest form?
  - $\textbf{F}~2\sqrt{18}~\text{cm}$ G 3√18 cm
- (H) 6√2 cm J 18√2 cm

### **Reading Strategies**

### 11-8 Use Models

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Use the four models below as a guide to multiplying and dividing radical



#### Complete the following.

1. Which method would be used to multiply  $(3 + \sqrt{7})^2$ ?

- 2. What does it mean to "rationalize the denominator"?
  - Rewrite the expression so there is no square root in the denominator.

Multiply. Write each product in simplest form.

- **3.**  $3\sqrt{6} \cdot 2\sqrt{3}$
- **4.**  $\sqrt{3}(5 + \sqrt{8})$
- **5.**  $(9 \sqrt{5}) (7 + \sqrt{5})$

- $18\sqrt{2}$
- $5\sqrt{3} + 2\sqrt{6}$

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 $58 + 2\sqrt{5}$ 

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Simplify each quotient by rationalizing the denominator.

- $\sqrt{x}$

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