REMARKABLE IDENTITIES

Square of a sum

$$(a + b)^2 = a^2 + 2ab + b^2$$

We say that $a^2 + 2ab + b^2$ is a perfect square trinomial.

Ex.: $(2x + 5y)^2 = (2x)^2 + 2(2x)(5y) + (5y)^2$
$= 4x^2 + 20xy + 25y^2$

Square of a difference

$$(a - b)^2 = a^2 - 2ab + b^2$$

We say that $a^2 - 2ab + b^2$ is a perfect square trinomial.

Ex.: $(3x - 2y)^2 = (3x)^2 - 2(3x)(2y) + (2y)^2$
$= 9x^2 - 12xy + 4y^2$

Product of a sum and a difference

$$(a + b)(a - b) = a^2 - b^2$$

The product of a sum by a difference is equal to a difference of two squares.

Ex.: $(3x + 5y)(3x - 5y) = (3x)^2 - (5y)^2$
$= 9x^2 - 25y^2$

1. a) Calculate the following products using the distributive property.

1. $(3x + 5)^2$ $9x^2 + 30x + 25$ 2. $(2x - 7)^2$ $4x^2 - 28x + 49$ 3. $(2x + 5)(2x - 5)$ $4x^2 - 25$

b) Calculate the following products using the appropriate remarkable identity.

1. $(3x + 5)^2$ $9x^2 + 30x + 25$ 2. $(2x - 7)^2$ $4x^2 - 28x + 49$ 3. $(2x + 5)(2x - 5)$ $4x^2 - 25$

2. Calculate the following products using the identity $(a + b)^2 = a^2 + 2ab + b^2$.

a) $(x + 5)^2$ $x^2 + 10x + 25$

b) $(3x + 4)^2$ $9x^2 + 24x + 16$

c) $(2x + 5y)^2$ $4x^2 + 20xy + 25y^2$

d) $\left(\frac{1}{2}x + 7\right)^2$ $\frac{1}{4}x^2 + 7x + 49$

e) $(3x^2y + xy^2)^2$ $9x^4y^2 + 6x^3y^3 + x^2y^4$

f) $\left(\frac{3}{4}x^2 + \frac{2}{9}y^2\right)^2$ $\frac{9}{16}x^4 + \frac{1}{3}x^2y^2 + \frac{4}{81}y^4$

3. Calculate the following products using the identity $(a - b)^2 = a^2 - 2ab + b^2$.

a) $(2x - 7)^2$ $4x^2 - 28x + 49$

b) $(4a - b)^2$ $16a^2 - 8ab + b^2$

c) $(3x^2 - 2x)^2$ $9x^4 - 12x^3 + 4x^2$

d) $(-3x - 4y)^2$ $9x^2 + 24xy + 16y^2$

e) $\left(\frac{3}{4}x - \frac{5}{6}\right)^2$ $\frac{9}{16}x^2 - \frac{5}{4}x + \frac{25}{36}$

f) $\left(\frac{2}{3}x^2 - \frac{3}{4}x\right)^2$ $\frac{4}{9}x^4 - x^3 + \frac{9}{16}x^2$

4. Calculate the following products using the identity $(a + b)(a - b) = a^2 - b^2$.

a) $(2x + 7)(2x - 7)$ $4x^2 - 49$

b) $(4x + 3y)(4x - 3y)$ $16x^2 - 9y^2$

c) $(2 - 3x)(2 + 3x)$ $4 - 9x^2$

d) $(-2x + 3y)(-2x - 3y)$ $4x^2 - 9y^2$

e) $\left(\frac{2}{3}x + \frac{3}{4}\right)\left(\frac{2}{3}x - \frac{3}{4}\right)$ $\frac{4}{9}x^2 - \frac{9}{16}$

f) $(5x^2y + 3x^3)(5x^2y - 3x^3)$ $25x^4y^2 - 9x^6$

5. Expand using the appropriate identity.

a) $(3x^2 - 2y)^2$ $9x^4 - 12x^2y + 4y^2$

b) $(2x^3 + 3y^2)(2x^3 - 3y^2)$ $4x^6 - 9y^4$

c) $(3x^4 + 2y^2)^2$ $9x^8 + 12x^4y^2 + 4y^4$

d) $(-3x^2 + 5x)^2$ $9x^4 - 30x^3 + 25x^2$

e) $(-2x - 3y)^2$ $4x^2 + 12xy + 9y^2$

f) $(5x^2 - 3y)(5x^2 + 3y)$ $25x^4 - 9y^2$

6. Expand, and simplify the resulting expressions.

a) $(3x + 2y)^2 + (2x - 3y)^2$ $13x^2 + 13y^2$

b) $(3x + 5y)^2 - (3x - 5y)^2$ $60xy$

c) $(2x + 3)(4x^2 + 9)(2x - 3)$ $16x^4 - 81$

d) $3x(2x - 3)^2 - 2x(3x + 2)^2$ $-6x^3 - 60x^2 + 19x$

e) $(x + 1)(x^2 - 1)(x - 1)$ $x^4 - 2x^2 + 1$

f) $(2x + 3)^2 + (2x - 3)^2 + (2x + 3)(2x - 3)$ $12x^2 + 9$

g) $(3x + 5)^2 - (3x - 5)^2 + (3x + 5)(3x - 5)$ $9x^2 + 60x - 25$

h) $(4x + 3y)^2 - (4x + 3y)(4x - 3y)$ $18y^2 + 24xy$

i) $(4x - 1)^2 - (3x + 5)^2$ $7x^2 - 38x - 24$

7. Use the remarkable identities to calculate mentally.

a) 101^2 $10\,201$ b) 99^2 $9\,801$ c) 101×99 $9\,999$

8. If x represents Karen's age, use a reduced polynomial to represent

a) the square of Karen's age 3 years from now; $x^2 + 6x + 9$

b) the square of Karen's age 5 years ago; $x^2 - 10x + 25$

c) the product of Karen's age 4 years from now with her age 4 years ago; $x^2 - 16$

d) the difference between the square of Karen's age 1 year from now and the square of her age 1 year ago; $4x$

9. Complete the following trinomials to obtain a perfect square trinomial.

a) $x^2 + 6x +$ 9

b) $4x^2 - 20x +$ 25

c) $9x^2$ $+ 12x + 4$

d) $9x^2$ $- 6x + 1$

e) $9x^2 +$ $24x$ $+ 16$

f) $16x^2 -$ $24x$ $+ 9$

g) $9x^4 + 12x^2y +$ $4y^2$

h) $4x^2$ $+ 12x^3 + 9x^4$

i) $25x^4 +$ $20x^2y^2$ $+ 4y^4$

10. Prove the following identities.

a) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

b) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

c) $(a + b)(a^2 - ab + b^2) = a^3 + b^3$

d) $(a - b)(a^2 + ab + b^2) = a^3 - b^3$