

Lesson 2

Operations on Polynomials

Date:

Chapter 1: Polynomials

Lesson 2: Operations on Polynomials

BEDMAS



Polynomials

REMINDER

In an algebraic expression, the **terms** are the different parts of the expression separated by a + or -.

Ex:

$$x^3 + 2x - 3$$

term term term

- **Monomial:** 1 term algebraic expression Ex:

- **Binomial:** 2-term algebraic expression Ex:

- **Trinomial:** 3-term algebraic expression Ex:

- **Polynomial:** Many-term algebraic expression Ex:

- **The degree of a monomial:** Sum of the exponents of its variables

Ex: $3x^3y^4z^1$ Degree: 8

- **The coefficient of a term:** The number at the front of a variable

Ex: $3x^2y$

- **LIKE TERMS:** Terms with variables raised to the same exponents

Ex: $4x^3 + 6x^3$ ~~$4x^3y + 3x^2y^3$~~

Add Subtract Multiply Divide

Don't forget BEMDAS

Adding Polynomials

Example:

$$(5x^2y + 3x - 9) + (4x^2y - 2x + 7)$$

$$9x^2y + x - 2$$

STEPS:

- 1) Drop the brackets if it's a plus sign at the front of the brackets.
- 2) Put like terms together.



Polynomials are in **simplest form** when they contain no like terms.

$$\cancel{x^2} + \cancel{2x} + 1 + \cancel{3x^2} - \cancel{4x}$$

when **simplified** becomes

$$4x^2 - 2x + 1$$

Like Terms	Like Terms	Not Like Terms
$2xy, 4xy, -3xy$	$4a^2b^3, 9a^2b^3$	$x^2, x, 2x^3, 4$

Subtracting Polynomials

Example:

$$(4x^2 + 11x - 13) - (-2x^2 - 7x + 8)$$

STEPS:

- 1) Change the sign of each term inside the brackets
if it's a minus sign at the front of the brackets.
- 2) Put like terms together.

$$4x^2 + 11x - 13 + 2x^2 + 7x - 8$$

$$6x^2 + 18x - 21$$

Multiplying a Polynomial by a Monomial

Example:

1) $5ab(3a^2 + 4b - 7)$

$$15a^3b + 20ab^2 - 35ab$$

STEPS:

- 1) Multiply each of the term of the polynomial by the monomial.

Reminder:

When multiplying variables, use the law of exponents.

2) $\frac{3}{4}x^2(\frac{2}{3}x - 8x^2)$

$$\frac{6}{12}x^3 - \frac{24}{4}x^4$$
$$\boxed{\frac{1}{2}x^3 - 6x^4}$$

3) $4x + 5x(3x - 4)$

$$4x + 15x^2 - 20x$$
$$\boxed{15x^2 - 16x}$$

Multiplying a BINOMIAL by a BINOMIAL

Example:

1) $(3x+5)(2x-4)$

First Outer Inner Last

$$6x^2 - 12x + 10x - 20$$

$$\boxed{6x^2 - 2x - 20}$$

example $(x+3)(x+2)$

F: $(x+3)(x+2)$

O: $(x+3)(x+2)$

I: $(x+3)(x+2)$

L: $(x+3)(x+2)$

2) $(x-2)^2$

$$(x-2)(x-2)$$

$$x^2 - 2x - 2x + 4$$

$$\boxed{x^2 - 4x + 4}$$

3) $(x+1)(x-3)(x-2)$

$$(x^2 - 3x + x - 3)(x-2)$$

$$(x^2 - 2x - 3)(x-2)$$

$$x^3 - \cancel{2}x^2 - \cancel{2}x^2 + 4x - 3x + 6$$

$$\boxed{x^3 - 4x^2 + x + 6}$$



Dividing a Polynomial by a Monomial

Example:

$$(12x^3 + 8x^2y - 3x) \div 3x$$

$$\cancel{12x^3} + \cancel{8x^2y} - 3x$$

$$4x^2 + \frac{8}{3}xy - 1$$

STEPS:

- 1) Divide each of the term of the polynomial by the monomial.

Reminder:

When dividing variables, use the law of exponents.

$$(-15x^3y^2 + 25x^2y - 55xy^4) \div -5xy^2$$

$$3x^2 - 5xy^{-1} + 11y^2$$

from workbook p.7-8 #17b)

$$P = \frac{2}{3}x^2 - \frac{3}{2}x + 1$$

$$Q = \frac{3}{2}x^2 + \frac{5}{6}x - \frac{1}{3}$$

$$R = \frac{3}{2}x - \frac{1}{6}$$

Détermine

$$P - Q + R$$

$$\left(\cancel{\frac{2}{3}x^2} - \cancel{\frac{3}{2}x} + 1 \right) - \left(\cancel{\frac{3}{2}x^2} + \cancel{\frac{5}{6}x} - \cancel{\frac{1}{3}} \right) + \left(\cancel{\frac{3}{2}x} - \cancel{\frac{1}{6}} \right)$$

$$-\frac{5}{6}x^2 - \frac{5}{6}x + \frac{7}{6}$$

Evaluate an algebraic expression:

Simply replace any variables with the prescribed value.

Ex. Evaluate if $P(x)$ if $x = -4$

$$\begin{aligned}P(x) &= 2x^2 - 3x + 7 \\&= 2(-4)^2 - 3(-4) + 7 \\&= 2(16) - 3(-4) + 7 \\&= 32 + 12 + 7 \\&= \boxed{51}\end{aligned}$$

Simplify:

a) $2x^3 - 5x^3 + 7x^3$ $4x^3$

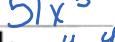
c) $\frac{3}{4}x^2 + \frac{2}{3}x^2 - x^2$ $\frac{5}{12}x^2$

b) $4x^2y - 6x^2y + x^2y$ $-x^2y$

d) $-\frac{2}{3}xy^2 + \frac{3}{4}xy^2 - \frac{5}{6}xy^2$ $-\frac{3}{4}xy^2$

What is the product?

a) $-3x^2 \times 4x^3$   $-12x^5$

c) $-17x^2 \times -3x$   $51x^3$

e) $3x^2y \times -5xy \times -2xy^2$   $30x^4y^4$

g) $\frac{3}{4}x^2y \times \frac{2}{5}xy^2 \times \frac{10}{9}x$   $\frac{1}{3}x^4y^3$

b) $2x^2y^3 \times -3xy^2$   $-6x^3y^5$

d) $-7x^2y \times 5x^2y^2$   $-35x^4y^3$

f) $20x^2y^2 \times -0.5x \times -1.2y^2$   $12x^3y^4$

h) $\frac{-3}{5}x^2y^3 \times \frac{2}{3}xy \times \frac{-5}{2}xy^2$   x^4y^6

what is the quotient?

a) $-12x^4 \div 3x^6$   $-4x^{-2}$

c) $18x^6y^4 \div 9x^2y^2$   $2x^4y^2$

e) $(-5x^3)^2 \div 10x^4$   $\frac{5}{2}x^2$

b) $18x^6 \div 12x^4$   $\frac{3}{2}x^2$

d) $-12x^2y^4 \div 6x^3y$   $-2x^{-1}y^3$

f) $(4x^2y^3)^3 \div (2xy^2)^4$   $4x^2y^3$

Simplify the following polynomials

$$-2x^2 - x + 1$$

a) $P(x) = 3x^2 + 2x - 5x^2 - 3x + 1$ _____

b) $P(x, y) = 3x^3y - 2xy^2 + 4x^3y - xy^2$ _____

$$7x^3y - 3xy^2$$

c) $P(z) = 4z^3 - 5z^2 + 8z^3 - z^2 + 4z - 5 + 6z^2 - 12z^3$ _____

$$4z - 5$$

d) $P(x) = \frac{3}{2}x^2 + 5x^3 - \frac{2}{3}x^2 - \frac{3}{2}x^3 + \frac{3}{4}x - \frac{5}{2}x$ _____

$$\frac{7}{2}x^3 - \frac{5}{6}x^2 - \frac{7}{4}x$$

Evaluate the following polynomials

a) $P(x) = 3x^2 + 5x$ for $x = -2$ _____

$$2$$

b) $P(x) = x^2 - 5x + 3$ for $x = 0$ _____

$$3$$

c) $P(x) = 3x^2 + 2x - 5$ for $x = -1.5$ _____

$$-1.25$$

d) $P(x) = 2x^2 - 7x - 15$ for $x = -\frac{3}{2}$ _____

$$0$$

Given $P = 3x^2 - 2x + 1$, $Q = -x^2 - 3x + 2$ and $R = -2x + 5$. Determine:

a) $P + Q + R$ _____

c) $P - Q - R$ _____

b) $P - Q + R$ _____

d) $-P + Q - R$ _____

a) $2x^2 - 7x + 8$

c) $4x^2 + 3x - 6$

b) $4x^2 - x + 4$

d) $-4x^2 + x - 4$

Perform the operations

a) $(4x^2 - 8x + 1) - (2x^2 - 3x + 5)$ $2x^2 - 5x - 4$

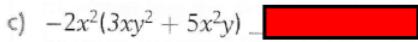
b) $(3x^2 - 2xy^2 + 3xy) + (2x^2 + 3x^2y - 5xy)$

c) $(3a^2b - 5ab^2) - (2a^2b + 3ab^2)$ $a^2b - 8ab^2$

b) $5x^2 + xy^2 - 2xy$

What is the product?

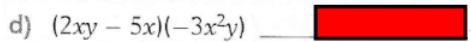
a) $3x^2(2x - 5)$ _____



b) $-3y(y^2 - 2y)$ _____



c) $-2x^2(3xy^2 + 5x^2y)$ _____



a) $6x^3 - 15x^2$

b) $-3y^3 + 6y^2$

c) $-6x^3y^2 - 10x^4y$

d) $-6x^3y^2 + 15x^3y$

Multiply the binomials:

a) $(x + 3)(x - 2)$ _____



b) $(x - 5)(3 - x)$ _____



c) $(2a + b)(3a - 2b)$ _____



d) $(5 - 2x)(3x - 4)$ _____

a) $x^2 + x - 6$

b) $-x^2 + 8x - 15$

c) $6a^2 - ab - 2b^2$

d) $-6x^2 + 23x - 20$

Perform the operations:

$$(3x + 5y)^2 - (3x - 5y)^2 \quad \underline{60xy}$$

$$(2x + 3)(4x^2 + 9)(2x - 3) \quad \underline{16x^4 - 81}$$

What makes Polynomial Operations difficult?

What are your strategies to combat this?

You can now do:

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