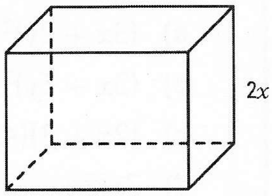
1.3 Polynomial division

ACTIVITY 1 Dividing a polynomial by a monomial

a) The prism on the right has the volume $V(x) = 12x^3 + 10x^2 - 2x$. Determine the area of the base knowing that the height of the prism is equal to $2x$.

$6x^2 + 5x - 1$



b) Explain how to divide a polynomial by a monomial.

Divide each term of the polynomial by the monomial.

DIVISION BY A MONOMIAL

- To divide a polynomial by a monomial, we divide each term of the polynomial by the monomial.

  Ex.: The quotient $Q(x)$ of the polynomial $A(x) = 18x^4 - 9x^3 + 12x^2$ by the monomial $B(x) = 6x^2$ is:

$$\begin{aligned} Q(x) &= A(x) \div B(x) \\ &= (18x^4 - 9x^3 + 12x^2) \div 6x^2 \\ &= (18x^4 \div 6x^2) - (9x^3 \div 6x^2) + (12x^2 \div 6x^2) \\ &= 3x^2 - \frac{3}{2}x + 2. \end{aligned}$$

- The quotient of two polynomials is not always a polynomial.

  Ex.: $(12x^3 - 6x) \div (3x^2) = 4x - 2x^{-1}$ which is not a polynomial.

1. Perform the following divisions.

a) $(24x^4 + 12x^3 - 18x^2) \div 6x^2$ ***$4x^2 + 2x - 3$***

b) $(36x^2y^4 + 27x^3y^2 - 9x^2y^2) \div 9x^2y$ ***$4y^3 + 3xy - y$***

c) $(3x^2 + 2x)(4x + 6) \div 2x$ ***$6x^2 + 13x + 6$***

d) $(3x + 6)^2 \div 3x$ ***$3x + 12 + 12x^{-1}$***

e) $(4x^2 + 2x)(4x^2 - 2x) \div 4x^2$ ***$4x^2 - 1$***

EUCLIDEAN DIVISION

- Consider the polynomials $A(x) = 6x^2 + 5x - 4$ and $B(x) = 3x - 2$.

  To divide $A(x)$ (the dividend) by $B(x)$ (the divisor), we proceed in the following manner to determine the quotient $Q(x)$ and the remainder $R(x)$:

When writing the division, make sure $A(x)$ and $B(x)$ are in decreasing order of the exponents.

$$\begin{array}{r|l} & Q(x) \\ \hline B(x) & A(x) \\ & R(x) \end{array}$$

1° Divide the term in the dividend with the highest degree ($6x^2$) by the term in the divisor with the highest degree ($3x$). We get ($2x$) which represents the term in the quotient $Q(x)$ with the highest degree.

$$\begin{array}{r|l} & 2x \\ \hline 3x-2 & 6x^2+5x-4 \end{array}$$

2° Calculate the product between the divisor ($3x-2$) and the 1st term 2x obtained in the first step. Align the resulting product ($6x^2-4x$) under the dividend.

$$\begin{array}{r|l} & 2x \\ \hline 3x-2 & 6x^2+5x-4 \\ & 6x^2-4x \end{array}$$

3° Calculate the first remainder by subtracting the product obtained in 2nd step ($6x^2-4x$) from the dividend. We then get ($9x-4$).

$$\begin{array}{r|l} & 2x \\ \hline 3x-2 & 6x^2+5x-4 \\ - & \underline{6x^2-4x} \\ & \qquad\quad 9x-4 \end{array}$$

Repeat the process...

Stop the division when the degree of the remainder is less than the degree of the divisor.

We, therefore, get the quotient $Q(x) = 2x + 3$ and the remainder $R(x) = 2$.

$$\begin{array}{r|l} & 2x+3 \leftarrow \text{quotient} \\ \hline 3x-2 & 6x^2+5x-4 \\ & \underline{6x^2-4x} \\ & \qquad\quad 9x-4 \\ - & \qquad\quad \underline{9x-6} \\ \text{remainder} \longrightarrow & \qquad\qquad\ 2 \end{array}$$

- The dividend $A(x)$, the divisor $B(x)$, the quotient $Q(x)$ and the remainder $R(x)$ verify the following Euclidean relation:

$$\boxed{A(x) = B(x) \cdot Q(x) + R(x)} \quad \text{where} \quad \deg R(x) < \deg B(x).$$

In fact, $6x^2 + 5x - 4 = (3x - 2)(2x + 3) + 2$.

2. Determine the quotient $Q(x)$ and the remainder $R(x)$ in the division of $A(x) = 2x^2 + 5x - 3$ by $B(x) = x - 1$. ***$Q(x) = 2x + 7$; $R(x) = 4$*** ***$(x - 1)(2x + 7) + 4 = 2x^2 + 5x - 3$***

3. In each of the following cases, determine the quotient $Q(x)$ and the remainder $R(x)$ in the division of $A(x)$ by $B(x)$.

a) $A(x) = 2x^2 - x - 6$; $B(x) = 2x + 3$ ***$Q(x) = x - 2$; $R(x) = 0$***

b) $A(x) = 3x^2 - 2x + 1$; $B(x) = x - 2$ ***$Q(x) = 3x + 4$; $R(x) = 9$***

c) $A(x) = 2x^3 + 3x^2 + 2x + 4$; $B(x) = x + 1$ ***$Q(x) = 2x^2 + x + 1$; $R(x) = 3$***

d) $A(x) = x^3 - 2x + 1$; $B(x) = x - 1$ ***$Q(x) = x^2 + x - 1$; $R(x) = 0$***

e) $A(x) = x^4 - 1$; $B(x) = x + 1$ ***$Q(x) = x^3 - x^2 + x - 1$; $R(x) = 0$***

f) $A(x) = x^3 + 27$; $B(x) = x + 3$ ***$Q(x) = x^2 - 3x + 9$; $R(x) = 0$***

ACTIVITY 2 Remainder in Euclidean division

a) Given $P(x) = 3x^2 - 5x + 1$.

1. Calculate $P(2)$. ***3***
2. Verify that the remainder in the division of $P(x)$ by $(x - 2)$ is equal to $P(2)$.
3. Calculate $P(-2)$. ***23***
4. Verify that the remainder in the division of $P(x)$ by $(x + 2)$ is equal to $P(-2)$.