

2. Factor the following polynomials.

a) $5x - 10$ $5(x - 2)$
 c) $4x^2 + 6x$ $2x(2x + 3)$
 e) $12a^2b + 18a^2b^2$ $6a^2b(2 + 3b)$
 g) $a^2 + ab + a$ $a(a + b + 1)$
 i) $24x^3y^2 - 16x^2y^3 + 28x^3y^4$ $4x^2y^2(6x - 4y + 7xy^2)$

b) $18x + 24y - 12z$ $6(3x + 4y - 2z)$
 d) $12x + x^2 - 5x^3$ $x(12 + x - 5x^2)$
 f) $-3x^4 + 6x^3 - 9x^2$ $-3x^2(x^2 - 2x + 3)$
 h) $x^4 - x^3y - x^2$ $x^2(x^2 - xy - 1)$
 j) $21x^3y^2z - 14x^2y^3z^2 + 28x^2y^2z^2$ $7x^2y^2z(3x - 2y + 4z)$

3. Factor the following polynomials.

a) $12x^3 - 16x^2$ $4x^2(3x - 4)$
 c) $18x^2y^3z^4 - 12xy^4z^3$ $6xy^3z^3(3xz - 2y)$
 e) $-14x^3 + 21x^2 - 7x$ $-7x(2x^2 - 3x + 1)$
 g) $2x(x + 1) + 3y(x + 1)$ $(x + 1)(2x + 3y)$

b) $6x^2y^3 + 4x^3y^2$ $2x^2y^2(3y + 2x)$
 d) $12x^2y^3 - 18x^3y^2 + 24x^2y^2$ $6x^2y^2(2y - 3x + 4)$
 f) $-25m^4n^3 + 50m^3n^4$ $-25m^3n^3(m - 2n)$
 h) $2(x - 3) - x(x - 3)$ $(x - 3)(2 - x)$

4. Factor the following polynomials.

a) $x(x + 2) + 5(x + 2)$ $(x + 2)(x + 5)$
 c) $a(b + c) - d(b + c)$ $(b + c)(a - d)$
 e) $(x + 3)(x + 2) + (x + 3)(x - 1)$ $(x + 3)(2x + 1)$
 g) $(x + y)^2 + x(x + y)$ $(x + y)(2x + y)$

b) $3(x - 2) - x(x - 2)$ $(x - 2)(3 - x)$
 d) $x(3 - y) + y(3 - y)$ $(3 - y)(x + y)$
 f) $(x + y)(x - 2) - (x + y)(2x - 3)$ $(x + y)(-x + 1)$
 h) $(x - y)^2 + (x - y)(x + y)$ $2x(x - y)$

5. Factor the following polynomials.

a) $x(x - 1) - 3(1 - x)$ $(x - 1)(x + 3)$
 c) $(x - 5)^2 - 2(5 - x)$ $(5 - x)(3 - x)$
 e) $(2x + 3y)(x + y) + (4x + 6y)(x - y)$ $(2x + 3y)(3x - y)$
 f) $(x + 1)(2x + 6) - (x - 2)(3x + 9)$ $(x + 3)(-x + 8)$

b) $x(x + 3) + 2(-x - 3)$ $(x + 3)(x - 2)$
 d) $(2x + 1)(2x - 1) + (1 - 2x)^2$ $4x(2x - 1)$

ACTIVITY 2 Factoring by grouping

a) The sum $ac + ad + bc + bd$ is composed of 4 terms. Each term is the product of two factors. Can we find a common factor to all 4 of these terms? No

b) Justify the steps which enable you to factor the sum $ac + ad + bc + bd$.

$$\begin{aligned} ac + ad + bc + bd &= (ac + ad) + (bc + bd) \quad \text{Addition is associative} \\ &= a(c + d) + b(c + d) \quad \text{Remove the common factor} \\ &= (c + d)(a + b) \quad \text{Remove the common factor} \end{aligned}$$

c) The rectangle on the right has an area of $A = 6x^2 + 4x + 9xy + 6y$.

What could the dimensions of this rectangle be?

$$\begin{aligned} (6x^2 + 4x) + (9xy + 6y) &= 2x(3x + 2) + 3y(3x + 2) \\ &= (3x + 2)(2x + 3y) \end{aligned}$$

$A = 6x^2 + 4x + 9xy + 6y$

?

The dimensions of the rectangle could be $(3x + 2)$ and $(2x + 3y)$.

FACTORING BY GROUPING

- Factoring by grouping is a method which enables you to factor polynomials by grouping the terms which contain a common factor.

You then remove the common factor in each of the groupings:

$$\begin{aligned} ac + ad + bc + bd &= (ac + ad) + (bc + bd) \\ &= a(c + d) + b(c + d) \\ &= (c + d)(a + b). \end{aligned}$$

Ex.: Factor the following expression using factoring by grouping.

$$\begin{aligned} P(x) &= \underbrace{9x^2 - 12xy^2}_{\text{Group the terms containing a common factor.}} + \underbrace{6xy - 8y^2}_{\text{Remove the common factor in each grouping.}} \\ &= 3x(3x - 4y^2) + 2y(3x - 4y) \quad \leftarrow \text{Remove the common factor a 2nd time.} \\ &= (3x - 4y^2)(3x + 2y) \end{aligned}$$

- 6.** Factor the following polynomials.

| | | | | | |
|----|-------------------------|--------------------------------------|----|---------------------------|---------------------------------------|
| a) | $x^2 + 5xy + 3x + 15y$ | <u>$(x + 3)(x + 5y)$</u> | b) | $2x^2 + 3xy - 10x - 15y$ | <u>$(x - 5)(2x + 3y)$</u> |
| c) | $6a^2 - 15a + 2ab - 5b$ | <u>$(3a + b)(2a - 5)$</u> | d) | $6x^2 - 8x - 9xy + 12y^2$ | <u>$(2x - 3y)(3x - 4)$</u> |
| e) | $10xy + 2x + 15y + 3$ | <u>$(2x + 3)(5y + 1)$</u> | f) | $x^3 - x^2 + x - 1$ | <u>$(x^2 + 1)(x - 1)$</u> |

- 7.** Factor the following polynomials.

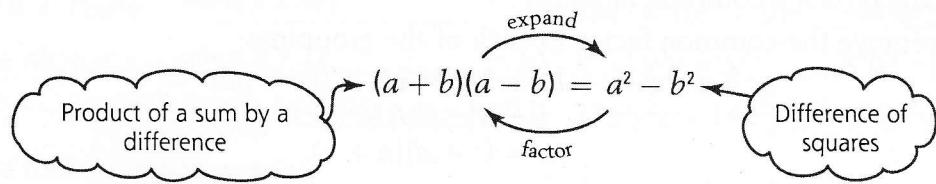
| | | |
|----|--|---|
| a) | $2x^2y + 3x^2 + 10y + 15$ | <u>$(x^2 + 5)(2y + 3)$</u> |
| b) | $15x^4y^2 + 35x^2y^2 - 9x^2 - 21$ | <u>$(3x^2 + 7)(5x^2y^2 - 3)$</u> |
| c) | $2x^3 + 4x^2y - 2x^2 - 4xy$ | <u>$2x(x - 1)(x + 2y)$</u> |
| d) | $3x^3y - 9x^3 + 6x^2y - 18x^2$ | <u>$3x^2(x + 2)(y - 3)$</u> |
| e) | $30x^4y - 10x^3y^2 + 15x^3y - 5x^2y^2$ | <u>$5x^2y(2x + 1)(3x - y)$</u> |
| f) | $2x^4 - 2x^3 + 6x^2 - 6x$ | <u>$2x(x^2 + 3)(x - 1)$</u> |

- 8.** Factor the following polynomials.

| | | |
|----|--|---|
| a) | $ax - ay + bx - by + cx - cy$ | <u>$(a + b + c)(x - y)$</u> |
| b) | $6ax - 3ay + 10bx - 5by - 4x + 2y$ | <u>$(3a + 5b - 2)(2x - y)$</u> |
| c) | $a^3 - 2ab + ac^2 - a^2b + 2b^2 - bc^2 + a^2c - 2bc + c^3$ | <u>$(a^2 - 2b + c^2)(a - b + c)$</u> |
| d) | $ab(x^2 + y^2) - xy(a^2 + b^2)$ | <u>$(ax - by)(bx - ay)$</u> |

ACTIVITY 3 Difference of squares

The following remarkable identity enables you to factor a difference of squares.



- a) Factor the following differences of squares.

1. $x^2 - 25$ _____ $(x + 5)(x - 5)$
2. $4x^2 - 9y^2$ _____ $(2x + 3y)(2x - 3y)$
3. $x^2 - 7$ _____ $(x + \sqrt{7})(x - \sqrt{7})$
4. $-x^2 + 9$ _____ $(3 + x)(3 - x)$
5. $(3x + 1)^2 - 4x^2$ _____ $[(3x + 1) + 2x][(3x + 1) - 2x] = (5x + 1)(x + 1)$

- b) The rectangle on the right has an area of $A = 16x^2 - 9$. What could be the possible dimensions of this rectangle?

$(4x + 3)$ and $(4x - 3)$

$A = 16x^2 - 9$

?

DIFFERENCE OF SQUARES

- A difference of squares is an algebraic expression of the form $a^2 - b^2$.
- Every difference of squares is factorable. You simply need to apply the remarkable identity:

$$a^2 - b^2 = (a + b)(a - b)$$

Ex.: Factor:

- $9x^2 - 4y^2 = (3x)^2 - (2y)^2$ ← Write in the form $a^2 - b^2$.
 $= (3x + 2y)(3x - 2y)$ ← Apply the remarkable identity.
- $(2x + 1)^2 - 36 = (2x + 1)^2 - 6^2$
 $= [(2x + 1) + 6][(2x + 1) - 6]$
 $= (2x + 7)(2x - 5)$
- $(3x + 5)^2 - (2x + 1)^2 = [(3x + 5) + (2x + 1)][(3x + 5) - (2x + 1)]$
 $= (5x + 6)(x + 4)$

- A sum of squares is not factorable.

9. Factor the following differences of squares.

| | |
|---|--|
| a) $x^2 - 25$ _____ $(x + 5)(x - 5)$ | b) $16x^2 - 9$ _____ $(4x + 3)(4x - 3)$ |
| c) $49x^2 - 36y^2$ _____ $(7x + 6y)(7x - 6y)$ | d) $36x^4 - 25y^6$ _____ $(6x^2 + 5y^3)(6x^2 - 5y^3)$ |
| e) $100 - x^2$ _____ $(10 + x)(10 - x)$ | f) $\frac{x^2}{16} - \frac{y^2}{9}$ _____ $\left(\frac{x}{4} + \frac{y}{3}\right)\left(\frac{x}{4} - \frac{y}{3}\right)$ |
| g) $x^2 - 3$ _____ $(x + \sqrt{3})(x - \sqrt{3})$ | h) $x^2 - 1$ _____ $(x + 1)(x - 1)$ |
| i) $16x^2 - \frac{1}{9}$ _____ $\left(4x + \frac{1}{3}\right)\left(4x - \frac{1}{3}\right)$ | j) $\frac{25}{16}x^2y^4 - \frac{4}{9}z^6$ _____ $\left(\frac{5}{4}xy^2 + \frac{2}{3}z^3\right)\left(\frac{5}{4}xy^2 - \frac{2}{3}z^3\right)$ |

10. Factor the following differences of squares.

a) $(3x - 1)^2 - 9$ _____
 $(3x + 2)(3x - 4)$

c) $(2x + 5)^2 - 16x^2$ _____
 $(6x + 5)(-2x + 5)$

e) $16x^2 - (3x + 2)^2$ _____
 $(7x + 2)(x - 2)$

g) $(x + 3)^2 - (2x + 5)^2$ _____
 $(3x + 8)(-x - 2)$

i) $4(x + 5)^2 - 1$ _____
 $(2x + 11)(2x + 9)$

b) $(x + 1)^2 - 4$ _____
 $(x + 3)(x - 1)$

d) $25x^2 - (2x - 5)^2$ _____
 $(7x - 5)(3x + 5)$

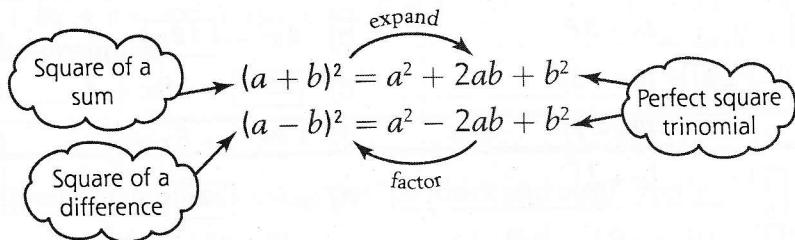
f) $36x^2 - (2 - x)^2$ _____
 $(5x + 2)(7x - 2)$

h) $(3x - 5y)^2 - (2x - 3y)^2$ _____
 $(5x - 8y)(x - 2y)$

j) $25(x - 3)^2 - 9(2x + 1)^2$ _____
 $(11x - 12)(-x - 18)$

ACTIVITY 4 Perfect square trinomials

The following remarkable identities enable you to factor a perfect square trinomial.



a) Use the remarkable identities to factor the following perfect square trinomials.

1. $x^2 + 6x + 9$ _____
 $(x + 3)^2$

2. $x^2 - 8x + 16$ _____
 $(x - 4)^2$

3. $4x^2 + 20x + 25$ _____
 $(2x + 5)^2$

4. $9x^2 - 12xy + 4y^2$ _____
 $(3x - 2y)^2$

b) The square on the right has an area of $A = 9x^2 - 12x + 4$.

What is the length of each side? _____
 $3x - 2$

$$A = 9x^2 - 12x + 4$$

c) 1. Explain why $4x^2 + 13x + 9$ is not a perfect square trinomial.

$4x^2 = (2x)^2; 9 = (3)^2$ but $13x \neq 2(2x)(3)$

2. Explain why $9x^2 + 30x + 25$ is a perfect square trinomial and factor it.

$9x^2 = (3x)^2; 25 = (5)^2$ and $30x = 2(3x)(5)$. $9x^2 + 30x + 25 = (3x + 5)^2$.

?

PERFECT SQUARE TRINOMIALS

- A perfect square trinomial is an algebraic expression of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$. A trinomial is a perfect square when the middle term is equal to twice the product of the square roots of the end terms.
- Every perfect square trinomial is factorable. You simply need to apply one of the remarkable identities below, depending on the sign of the middle term.

$$a^2 + 2ab + b^2 = (a + b)^2$$

or

$$a^2 - 2ab + b^2 = (a - b)^2$$

Ex.: Factor:

- $4x^2 + 12x + 9 = (2x)^2 + 2(2x)(3) + 3^2$
 $= (2x + 3)^2$
- $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$
 $= (2x - 3)^2$

← We write in the form: $a^2 + 2ab + b^2$.

← We apply the remarkable identity.

← We write in the form: $a^2 - 2ab + b^2$.

← We apply the remarkable identity.

11. Factor the following perfect square trinomials.

a) $x^2 + 10x + 25 \quad (x + 5)^2$

c) $4x^2 + 12xy + 9y^2 \quad (2x + 3y)^2$

e) $9x^4 - 30x^2 + 25 \quad (3x^2 - 5)^2$

g) $x^2 - x + \frac{1}{4} \quad \left(x - \frac{1}{2}\right)^2$

b) $x^2 - 14x + 49 \quad (x - 7)^2$

d) $25x^2 - 20xy + 4y^2 \quad (5x - 2y)^2$

f) $25x^4 + 30x^2y^3 + 9y^6 \quad (5x^2 + 3y^3)^2$

h) $\frac{9}{16}x^2 + x + \frac{4}{9} \quad \left(\frac{3}{4}x + \frac{2}{3}\right)^2$

12. Explain why the following trinomials are not perfect squares.

a) $4x^2 + 6x + 9 \quad 6x \neq 2 \times 2x \times 3$

b) $4x^2 + 12x - 9 \quad \text{The term } -9 \text{ is negative.}$

c) $-4x^2 + 12x + 9 \quad \text{The term } -4x^2 \text{ is negative.}$

d) $9x^2 - 15x + 25 \quad 15x \neq 2 \times 3x \times 5$

13. Complete the trinomials to obtain perfect square trinomials and factor them.

a) $x^2 + \boxed{6x} + 9 \quad (x + 3)^2$

c) $9x^2 + 30x + \boxed{25} \quad (3x + 5)^2$

e) $4x^2 - 28x + \boxed{49} \quad (2x - 7)^2$

g) $x^2 + \frac{2}{3}x + \boxed{\frac{1}{9}} \quad \left(x + \frac{1}{3}\right)^2$

b) $4x^2 - \boxed{12x} + 9 \quad (2x - 3)^2$

d) $\boxed{25x^2} + 20x + 4 \quad (5x + 2)^2$

f) $\boxed{9x^2} - 6x + 1 \quad (3x - 1)^2$

h) $x^2 - \boxed{7x} + \frac{49}{4} \quad \left(x - \frac{7}{2}\right)^2$

14. Complete the factoring of the trinomials of the form $x^2 + bx + c$ (the coefficient of x^2 is equal to 1).

a) $x^2 + 7x + 12 = (x \underline{+ 3})(x \underline{+ 4})$

b) $x^2 - 7x + 10 = (x \underline{- 2})(x \underline{- 5})$

c) $x^2 + 2x - 15 = (x \underline{+ 5})(x \underline{- 3})$

d) $x^2 - 5x - 14 = (x \underline{+ 2})(x \underline{- 7})$

e) $x^2 + 14x + 48 = (x \underline{+ 8})(x \underline{+ 6})$

f) $x^2 - 15x + 36 = (x \underline{- 12})(x \underline{- 3})$

g) $x^2 - 8x - 33 = (x \underline{- 11})(x \underline{+ 3})$

h) $x^2 + 2x - 63 = (x \underline{+ 9})(x \underline{- 7})$

ACTIVITY 5 Second degree trinomials $ax^2 + bx + c$

a) The rectangle on the right has an area of $A = x^2 + 10x + 16$.

1. Explain why $x^2 + 10x + 16$ is not a perfect square trinomial.

$10x \neq 2(x)(4)$

$A = x^2 + 10x + 16$

?

2. Find a method for factoring this trinomial. What could be the possible dimensions of this rectangle?

$x^2 + 10x + 16 = (x + 2)(x + 8)$. Possible dimensions: $(x + 2)$ and $(x + 8)$

b) The trinomial $2x^2 + 9x + 20$ is not a perfect square. Find a method for factoring this trinomial and factor it.

$$\begin{aligned} 2x^2 + 9x + 10 &= 2x^2 + 4x + 5x + 10 \\ &= 2x(x + 2) + 5(x + 2) \\ &= (x + 2)(2x + 5) \end{aligned}$$

SECOND DEGREE TRINOMIALS: $ax^2 + bx + c$

- The “product and sum” method enables you to factor a second degree trinomial. Let us illustrate this method by factoring $P(x) = 2x^2 + 7x + 6$.

1. Identify the coefficients a , b and c :
2. Find two integers m and n such that

$$\begin{cases} m \cdot n = ac & \leftarrow \text{product of the end coefficients} \\ m + n = b & \leftarrow \text{middle coefficient.} \end{cases}$$
3. Write: $ax^2 + bx + c = ax^2 + mx + nx + c$
and factor by grouping.

1. $a = 2; b = 7; c = 6$
2. $\begin{cases} mn = 12 \\ m + n = 7 \end{cases}$
 $m = 4, n = 3$
3. $2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$
 $= 2x(x + 2) + 3(x + 2)$
 $= (x + 2)(2x + 3)$

- 15.** Factor the following trinomials using the “product and sum” method.

- | | | | |
|----------------------|--------------------------------------|----------------------|--------------------------------------|
| a) $2x^2 + 9x + 4$ | <u>$(2x + 1)(x + 4)$</u> | b) $6x^2 - 19x + 10$ | <u>$(3x - 2)(2x - 5)$</u> |
| c) $4x^2 - 5x - 21$ | <u>$(4x + 7)(x - 3)$</u> | d) $5x^2 - 32x - 21$ | <u>$(5x + 3)(x - 7)$</u> |
| e) $12x^2 + 13x + 3$ | <u>$(3x + 1)(4x + 3)$</u> | f) $16x^2 - 26x + 3$ | <u>$(8x - 1)(2x - 3)$</u> |
| g) $6x^2 + 11x - 10$ | <u>$(3x - 2)(2x + 5)$</u> | h) $8x^2 + 2x - 15$ | <u>$(2x + 3)(4x - 5)$</u> |
| i) $x^2 + 10x + 24$ | <u>$(x + 6)(x + 4)$</u> | j) $x^2 - 11x + 30$ | <u>$(x - 6)(x - 5)$</u> |

- 16.** Factor the following trinomials.

- | | | | |
|---------------------|--------------------------------------|----------------------|--------------------------------------|
| a) $x^2 - 10x + 21$ | <u>$(x - 3)(x - 7)$</u> | b) $x^2 - 5x - 14$ | <u>$(x - 7)(x + 2)$</u> |
| c) $x^2 - 7x + 12$ | <u>$(x - 3)(x - 4)$</u> | d) $x^2 - 9x + 20$ | <u>$(x - 5)(x - 4)$</u> |
| e) $2x^2 + 7x + 3$ | <u>$(2x + 1)(x + 3)$</u> | f) $3x^2 + 5x - 2$ | <u>$(3x - 1)(x + 2)$</u> |
| g) $6x^2 + x - 2$ | <u>$(2x - 1)(3x + 2)$</u> | h) $10x^2 - 19x + 6$ | <u>$(5x - 2)(2x - 3)$</u> |

- 17.** Factor the following trinomials.

- | | | | |
|----------------------|--------------------------------------|---------------------|-------------------------------------|
| a) $x^2 + 8x + 15$ | <u>$(x + 3)(x + 5)$</u> | b) $x^2 - 8x + 15$ | <u>$(x - 3)(x - 5)$</u> |
| c) $x^2 + 5x - 14$ | <u>$(x + 7)(x - 2)$</u> | d) $x^2 + 9x + 14$ | <u>$(x + 7)(x + 2)$</u> |
| e) $6x^2 + 19x + 15$ | <u>$(2x + 3)(3x + 5)$</u> | f) $2x^2 - 7x - 15$ | <u>$(2x + 3)(x - 5)$</u> |
| g) $3x^2 - x - 4$ | <u>$(3x - 4)(x + 1)$</u> | h) $5x^2 - 17x + 6$ | <u>$(5x - 2)(x - 3)$</u> |

ACTIVITY 6 Multi-step factoring

Explain the steps in factoring the following polynomials.

a) $2x^3 - 18x = 2x(x^2 - 9)$ *Remove the common factor*
 $= 2x(x + 3)(x - 3)$ *Difference of two squares*

b) $12x^3 - 12x^2 + 3x = 3x(4x^2 - 4x + 1)$ Remove the common factor
 $= 3x(2x - 1)^2$ Perfect square trinomial

c) $4x^3 - 4x^2 - 8x = 4x(x^2 - x - 2)$ Remove the common factor
 $= 4x(x + 1)(x - 2)$ Non-perfect square trinomial

d) $x^4 - 16 = (x^2 + 4)(x^2 - 4)$ Difference of two squares
 $= (x^2 + 4)(x + 2)(x - 2)$ Difference of two squares

e) $x^4 - 8x^2 + 16 = (x^2 - 4)^2$ Perfect square trinomial
 $= [(x + 2)(x - 2)]^2$ Difference of two squares
 $= (x + 2)^2(x - 2)^2$ Property $(ab)^2 = a^2b^2$

MULTI-STEP FACTORING

Many steps are sometimes necessary to completely factor a polynomial.

Ex.: $2x^3 - 18x = 2x(x^2 - 9)$ ← remove the common factor
 $= 2x(x + 3)(x - 3)$ ← difference of squares

$$\begin{aligned} 4x(2x + 3) + 4x^2 - 9 &= 4x(2x + 3) + (2x + 3)(2x - 3) && \leftarrow \text{difference of squares} \\ &= (2x + 3)[4x + (2x - 3)] && \leftarrow \text{remove the common factor} \\ &= (2x + 3)(6x - 3) && \leftarrow \text{reduce} \\ &= (2x + 3) \cdot 3(2x - 1) && \leftarrow \text{remove the common factor} \\ &= 3(2x + 3)(2x - 1) && \leftarrow \text{commutative property of multiplication} \end{aligned}$$

18. Completely factor the following polynomials.

a) $x^3 + 3x^2 + 2x = \underline{x(x + 1)(x + 2)}$

b) $x^3 - 2x^2 + x = \underline{x(x - 1)^2}$

c) $x^3 - 16x = \underline{x(x + 4)(x - 4)}$

d) $x^4 + 2x^2 + 1 = \underline{(x + 1)^2(x - 1)^2}$

e) $x^4 - 1 = \underline{(x^2 + 1)(x + 1)(x - 1)}$

19. Completely factor the following polynomials.

a) $6x^4 + 9x^3 + 3x^2 = \underline{3x^2(x + 1)(2x + 1)}$

b) $3x^3 - 12x = \underline{3x(x + 2)(x - 2)}$

c) $16x^4 - 8x^2 + 1 = \underline{(2x + 1)^2 \cdot (2x - 1)^2}$

d) $-x^2 + 6x - 9 = \underline{-(x - 3)^2}$

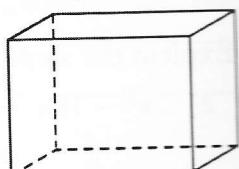
e) $(x^2 - 1) + (x - 1)^2 = \underline{2x(x - 1)}$

20. The prism on the right has a volume of $V = 2x^3 + 6x^2 + 4x$.

What could the dimensions of this prism be?

$$\begin{aligned} 2x^3 + 6x^2 + 4x &= 2x(x^2 + 3x + 2) \\ &= 2x(x + 1)(x + 2) \end{aligned}$$

The dimensions are: $2x$, $(x + 1)$ and $(x + 2)$.



- 21.** The area of a rectangle is expressed by the polynomial $A(x) = 6x^2 + 17x + 12$. What could be the perimeter of this rectangle?

Perimeter: $10x + 14$; possible dimensions: $(3x + 4)$ and $(2x + 3)$

- 22.** The area of a square is expressed by the polynomial $9x^2 + 12x + 4$. What is the perimeter of this square?

$12x + 8$

- 23.** The volume of a right rectangular prism is expressed by $V(x) = x^3 + 2x^2 - x - 2$. What could the dimensions of the prism be?

$(x + 1)$, $(x - 1)$ and $(x + 2)$

- 24.** The total area of a cube is expressed by $A(x) = 24x^2 + 24x + 6$. What is the volume of this cube?

$8x^3 + 12x^2 + 6x + 1$

- 25.** The volume of a right rectangular prism is expressed by $V(x) = x^3 + 4x^2 + x - 6$. If $(x + 3)$ represents the height of the prism, find two binomials that could express the dimensions of the prism's base.

$(x + 2)$ and $(x - 1)$

- 26.** A sum of two cubes and a difference of two cubes can be factored.

a) Show that

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$. Expand the right side.

2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. Expand the right side.

b) Factor

1. $x^3 + 64$ $(x + 2)(x^2 - 2x + 4)$

2. $8x^3 - 27$ $(2x - 3)(4x^2 + 6x + 9)$

3. $27x^3 - 8y^3$ $(3x - 2y)(9x^2 + 6xy + 4y^2)$