Trig Circle Review 1

1. Determine the exact values of the following.

a) $\cot 30°$ b) $\sec\frac{37\pi}{6}$ c) $\frac{\tan 45°}{\csc 60°}$

d) $\tan\frac{-33\pi}{4}$ e) $\sin\frac{67\pi}{3}$ f) $\csc\left(-\frac{113\pi}{3}\right)$

2. In which quadrant would you find ...

a) $P(-1200°)$ b) $-\frac{73\pi}{11}$ c) $\frac{93\pi}{17}$

d) $492°$ c) $P\left(\frac{28\pi}{5}\right)$ d) $2.5\ rad$

3. If $P(\theta)=\left(-\frac{5}{7},\frac{2\sqrt{6}}{7}\right)$, determine the <u>exact</u> values of ...

a) $\sec\theta$ b) $\cot\theta$ c) $\cos\theta$

4. Find the coterminal angle within the restriction $0\le\theta\le 2\pi$ that corresponds to ...

a) $480°$ b) $\frac{-17\pi}{6}$ c) $\frac{115\pi}{4}$

d) $-\frac{39\pi}{2}$ e) $\frac{61\pi}{3}$ d) $-513°$

5. Given $\csc x = 2.366$, determine ...

a) $\sin x$ b) $\cot x$

6. Determine the area of the circle whose arc length is $30cm$ and central angle $\theta=\frac{3\pi}{8}$.

7. Determine the value of x, if ...

a) $\sec 40°=\frac{x}{3}$ b) $\frac{\cos x}{\sin x}=2.5$ c) $\csc 51°=x$

8. Given $\cos\theta=-\frac{3}{5}$, determine the possible value(s) of $\tan\theta$, if $\frac{\pi}{2}\le\theta\le\frac{3\pi}{2}$.

9. Given a trig point $P(\theta)=\left(\frac{m}{n},-\frac{4}{5}\right)$, determine the exact value of $\cot\theta$.

10. Determine the coordinates of the following trigonometric points (exact values where possible).

a) $P\left(\frac{\pi}{5}\right)$ b) $P\left(\frac{29\pi}{4}\right)$ c) $P(122°)$

d) $P\left(-\frac{83\pi}{3}\right)$ e) $P\left(\frac{107\pi}{6}\right)$ f) $P\left(\frac{119\pi}{7}\right)$

11. Eleanor rode the carousel at a fair. The radius of the circular path she traveled was $2.1m$. Eleanor's speed was 1.5 m/s and the ride lasted for $2\frac{1}{2}$ minutes. What is the measure of the central angle that corresponds to Eleanor's ride....

a) in radians? b) in degrees?

12. Prove the following trigonometric identities

a) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

b) $\dfrac{\cot^2 x}{1+\dfrac{1}{\tan^2 x}} = \cos^2 x$

c) $\dfrac{\sec^2\varphi - 1}{1+\cot^2\varphi} * \dfrac{\csc^2\varphi}{\tan\varphi} * \csc\varphi = \sec\varphi$

d) $\dfrac{2\cos^2 x - \cos x - 1}{\cos x - 1} = 2\cos x - 1$

e) $\sin^2\varphi\left(1+\cot^2\varphi\right) + \cos^2\varphi\left(1+\tan^2\varphi\right) = 2$

f) $\left(1+\tan^2\theta\right)\left(1-\cos^2\theta\right) = \sec^2\theta - 1$

g) $\dfrac{\cos^2\alpha}{1-\sin\alpha} = 1+\sin\alpha$

h) $\dfrac{\sec^2\theta\cot\theta}{\csc^2\theta} = \tan\theta$

i) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

j) $\csc^2\alpha - \cot^2\alpha = 1$

13. Solve the following trigonometric equations.

a) $\tan x = \sec x$

b) $\cos\delta + 2 = 3\cos\delta$

c) $2\sin\alpha\cos\alpha = \sqrt{2}\cos\alpha$

d) $\sin^2\sigma = 3\cos^2\sigma$

e) $3\cot\beta + 5\sec\beta = 4$

f) $\sec^2 x + 3\tan x - 11 = 0$

Trig Circle Reveiw 1 Answers

1. a) $\cot 30^\circ = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

b) $\sec \frac{37\pi}{6} = \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c) $\frac{\tan 45^\circ}{\csc 60^\circ} = \frac{\left(\sqrt{2}/2 \div \sqrt{3}/2\right)}{2/\sqrt{3}} = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}$

d) $\tan\left(-\frac{33\pi}{4}\right) = \tan -\frac{\pi}{4} = \tan \frac{7\pi}{4} = \left(\frac{-\sqrt{2}/2}{\sqrt{2}/2}\right) = -1$

e) $\sin \frac{67\pi}{3} = \sin \pi/3 = \sqrt{3}/2$

f) $\csc\left(-\frac{113\pi}{3}\right) = \csc \frac{\pi}{3} = \frac{2}{\sqrt{3}} \Rightarrow \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3}$

2. a) $P(-1200^\circ) \Rightarrow$ Quad 3

b) $-\frac{73}{11}\pi \Rightarrow 3$

c) $\frac{93\pi}{17} \Rightarrow 3$

d) $492^\circ \Rightarrow 2$

e) $P\left(\frac{28\pi}{5}\right) \Rightarrow 4$

f) 2.5 Rad $\Rightarrow 2$

3. a) $\sec\theta = -\frac{7}{5}$ b) $\cot\theta = \frac{-5/7}{2\sqrt{6}/7}$ c) $\cos\theta = -\frac{5}{7}$

$-5/7 \times 7/2\sqrt{6}$

$-5/2\sqrt{6}$

$\frac{-5}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$\frac{-5\sqrt{6}}{12}$

4. a) $120°$ b) $\frac{7\pi}{6}$ c) $\frac{3\pi}{4}$

d) $\pi/2$ e) $\pi/3$ f) $207°$

5. $\csc x = 2.366$

a) $\sin x = \frac{1}{2.366} \approx 0.423$

$x = \sin^{-1}(0.423)$

$x = 0.436$ Rad

b) $\cot x = \frac{1}{\tan x}$

$= \frac{1}{\tan 0.436}$

$= 1/0.466$

≈ 2.14

6. $L = \theta r$ $L = 30$ $\theta = \frac{3\pi}{8}$

$r = L/\theta$

$r = 30/\frac{3\pi}{8}$

$r = 25.46$ cm

$A = \pi r^2 = \pi(25.46)^2$

$= 2037.18$ cm^2

7. a) $\sec 40° = \frac{x}{3}$

$x = 3 \sec 40°$

$x = \frac{3}{\cos 40°}$

$x = 3.92$

b) $\frac{\cos x}{\sin x} = 2.5$

$\frac{\sin x}{\cos x} = \frac{1}{2.5}$

$\tan x = 0.4$

$x = \tan^{-1} 0.4$

$x = 0.38$ Rad

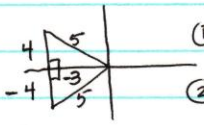
c) $\csc 51° = x$

$\frac{1}{\sin 51°} = x$

$\frac{1}{0.777} = x$

$1.29 = x$

8. $\cos\theta = -\frac{3}{5}$ $\quad \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ (quadrants 2 & 3)



① $\sin\theta = 4/5$

② $\sin\theta = -4/5$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{-3/5} = \frac{4}{5} \times -\frac{5}{3}$

$= -4/3$

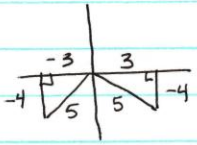
or $= \frac{-4/5}{-3/5} = +4/3$

9. $P(\theta) = \left(\frac{m}{n}, -\frac{4}{5}\right)$ $\quad \therefore \sin\theta = -\frac{4}{5}$ → in quadrant 3 or 4

$m = 3$ or -3

$n = 5$

$\cos\theta = \frac{m}{n} = -\frac{3}{5}$ or $\frac{3}{5}$



$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \frac{-3/5}{-4/5} \Rightarrow -\frac{3}{5} \times \frac{5}{-4} = \frac{-3}{-4} = \frac{3}{4}$

or $\frac{3/5}{-4/5} \Rightarrow -\frac{3}{4}$

10. a) $P\left(\frac{\pi}{5}\right) = \left(\cos\frac{\pi}{5}, \sin\frac{\pi}{5}\right) \approx (0.809, 0.588)$

b) $P\left(\frac{29\pi}{4}\right) = P\left(\frac{28\pi}{4} + \frac{\pi}{4}\right) = P\left(7\pi + \frac{\pi}{4}\right) = P\left(\frac{5\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

c) $P(122°) = (\cos 122°, \sin 122°) \approx (-0.53, 0.848)$

d) $P\left(-\frac{83\pi}{3}\right) = P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

e) $P\left(\frac{107\pi}{6}\right) = P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

f) $P\left(\frac{119\pi}{7}\right) = P(17\pi) = P(\pi) = (-1, 0)$

11. $r = 2.1\text{ m}$

$L = 1.5\text{ m/s} \times \frac{60\text{ seconds}}{\text{minute}} \times 2.5\text{ minutes}$

$= 225\text{ m}$

$L = \theta r$

$\theta = L/r$

$\theta = \frac{225}{2.1}$

a) $\theta = 107.14$ Radians

b) $\frac{n^\circ}{180} = \frac{107.14}{\pi}$

$n^\circ = \frac{107.14\,(180)}{\pi}$

$n = 6\,138.83^\circ$

12. a) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

$(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) = 2\sin^2\theta - 1$

$1\,(\sin^2\theta - (1 - \sin^2\theta)) = 2\sin^2\theta - 1$

$\sin^2\theta - 1 + \sin^2\theta = 2\sin^2\theta - 1$

$2\sin^2\theta - 1 = 2\sin^2\theta - 1$

f) $(1 + \tan^2\theta)(1 - \cos^2\theta) = \sec^2\theta - 1$

$(\sec^2\theta)(\sin^2\theta) = \sec^2\theta - 1$

$\frac{1}{\cos^2\theta} \cdot \sin^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta - 1 = \sec^2\theta - 1$

b) $\dfrac{\cot^2 x}{1+\dfrac{1}{\tan^2 x}} = \cos^2 x$

$\dfrac{\cot^2 x}{1+\cot^2 x} = \cos^2 x$

$\dfrac{\cot^2 x}{\csc^2 x} = \cos^2 x$

$\dfrac{\dfrac{\cos^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}} = \cos^2 x$

$\dfrac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \dfrac{\cancel{\sin^2 x}}{1} = \cos^2 x$

$\cos^2 x = \cos^2 x$

c) $\dfrac{\sec^2\varphi - 1}{1+\cot^2\varphi} \cdot \dfrac{\csc^2\varphi}{\tan\varphi} \cdot \csc\varphi = \sec\varphi$

$\dfrac{\tan^2\varphi}{\csc^2\varphi} \cdot \dfrac{\csc^2\varphi}{\tan\varphi} \cdot \csc\varphi = \sec\varphi$

$\tan\varphi \cdot \csc\varphi = \sec\varphi$

$\dfrac{\cancel{\sin\varphi}}{\cos\varphi} \cdot \dfrac{1}{\cancel{\sin\varphi}} = \sec\varphi$

$\dfrac{1}{\cos\varphi} = \sec\varphi$

$\sec\varphi = \sec\varphi$

d) $\dfrac{2\cos^2 x - \cos x - 1}{\cos x - 1} = 2\cos x - 1$

$\dfrac{(2\cos x + 1)\cancel{(\cos x - 1)}}{\cancel{\cos x - 1}} = 2\cos x - 1$

$2\cos x + 1 = 2\cos x - 1$

e) $\sin^2\varphi(1+\cot^2\varphi) + \cos^2\varphi(1+\tan^2\varphi) = 2$

$\sin^2\varphi \cdot \csc^2\varphi + \cos^2\varphi \cdot \sec^2 = 2$

$1 + 1 = 2$

$2 = 2$

g) $\dfrac{\cos^2\alpha}{1-\sin\alpha} = 1 + \sin\alpha$

$\dfrac{1-\sin^2\alpha}{1-\sin\alpha} = 1 + \sin\alpha$

$\dfrac{(1+\sin\alpha)\cancel{(1-\sin\alpha)}}{\cancel{1-\sin\alpha}} = 1 + \sin\alpha$

$1 + \sin\alpha = 1 + \sin\alpha$

h) $\dfrac{\sec^2\theta \cot\theta}{\csc^2\theta} = \tan\theta$

$\dfrac{\dfrac{1}{\cos^2\theta} \cdot \dfrac{\cancel{\cos\theta}}{\sin\theta}}{1/\sin^2\theta} = \tan\theta$

$\dfrac{1}{\cos\theta\cancel{\sin\theta}} \times \dfrac{\sin^{\cancel{2}}\theta}{1} = \tan\theta$

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan\theta = \tan\theta$

i) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1 - 2\cos^2 x$

$\sin^2 x - \cos^2 x = 1 - 2\cos^2 x$

$1 - \cos^2 x - \cos^2 x = 1 - 2\cos^2 x$

$1 - 2\cos^2 x = 1 - 2\cos^2 x$

j) $\csc^2 \alpha - \cot^2 \alpha = 1$

$\cot^2 \alpha + 1 - \cot^2 \alpha = 1$

$1 = 1$

13. a) $\tan x = \sec x$

$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$

$\frac{\sin x - 1}{\cos x} = 0$

$\sin x - 1 = 0$ $\quad \cos x \neq 0$

$\sin x = 1$ $\quad x \neq \{\pi/2, 3\pi/2\}$

$x = \{\pi/2\}$

∴ No solution

b) $\cos\delta + 2 = 3\cos\delta$

$2 = 2\cos\delta$

$1 = \cos\delta$

$\delta = \{0\}$

c) $2\sin\alpha\cos\alpha = \sqrt{2}\cos\alpha$

$2\sin\alpha\cos\alpha - \sqrt{2}\cos\alpha = 0$

$\cos\alpha(2\sin\alpha - \sqrt{2}) = 0$

$\cos\alpha = 0$ $\quad 2\sin\alpha - \sqrt{2} = 0$

$\alpha = \{\pi/2, 3\pi/2\}$ $\quad 2\sin\alpha = \sqrt{2}$

$\sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = \{\pi/4, 3\pi/4\}$

∴ $\alpha = \{\pi/4, \pi/2, 3\pi/4, 3\pi/2\}$

d) $\sin^2\sigma = 3\cos^2\sigma$

$0 = 3\cos^2\sigma - \sin^2\sigma$

$0 = 3\cos^2\sigma - (1 - \cos^2\sigma)$

$0 = 4\cos^2\sigma - 1$

$1 = 4\cos^2\sigma$

$1/4 = \cos^2\sigma$

$\pm 1/2 = \cos\sigma$

$\sigma = \{\pi/3, 2\pi/3, 4\pi/3, 5\pi/3\}$

13 e) ERROR

f) $\sec^2 x + 3\tan x - 11 = 0$

$1 + \tan^2 x + 3\tan x - 11 = 0$

$\tan^2 x + 3\tan x - 10 = 0$

$(\tan x - 2)(\tan x + 5) = 0$

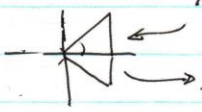
$\tan x - 2 = 0$ or $\tan x + 5 = 0$

$\tan x = 2$ $\qquad$ $\tan x = -5$

$x = \tan^{-1}(2)$ $\qquad$ $x = \tan^{-1}(-5)$

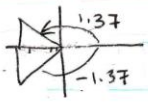
$x \doteq 1.11$ rad $\qquad$ $x = -1.37$

$= 4.91$



$x = 2\pi - 1.11$

$x \doteq 4.71$



$x = 1.37$

$x \approx \{1.11, 1.37, 4.71, 4.91\}$