

**2.** Knowing that  $a = 30^\circ$ , verify that

a)  $\sin 2a = 2 \sin a \cos a$

$$\underline{\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ}$$

$$\underline{\frac{\sqrt{3}}{2} = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}}$$

c)  $\cos 2a = 2 \cos^2 a - 1$

$$\underline{\cos 60^\circ = 2 \cos^2 30^\circ - 1}$$

$$\underline{\frac{1}{2} = 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 - 1}$$

b)  $\cos 2a = \cos^2 a - \sin^2 a$

$$\underline{\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ}$$

$$\underline{\frac{1}{2} = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

d)  $\cos 2a = 1 - 2 \sin^2 a$

$$\underline{\cos 60^\circ = 1 - 2 \sin^2 30^\circ}$$

$$\underline{\frac{1}{2} = 1 - 2 \left(\frac{1}{2}\right)^2}$$

**3.** Use the addition formulas to simplify

a)  $\sin(\pi + x) = \underline{\sin \pi \cos x + \sin x \cos \pi = -\sin x}$  b)  $\cos(\pi + x) = \underline{\cos \pi \cos x - \sin \pi \sin x = -\cos x}$

c)  $\sin\left(\frac{\pi}{2} + x\right) = \underline{\sin \frac{\pi}{2} \cos x + \sin x \cos \frac{\pi}{2} = \cos x}$  d)  $\cos\left(\frac{\pi}{2} + x\right) = \underline{\cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x = -\sin x}$

**4.** Knowing that  $\sin a = \frac{3}{5}$  and  $\sin b = \frac{12}{13}$  and that  $0 \leq a \leq \frac{\pi}{2}$  and  $0 \leq b \leq 2$ , calculate

a)  $\sin(a + b)$

$$\underline{\sin a = \frac{3}{5} \text{ and } 0 \leq a \leq \frac{\pi}{2} \Rightarrow \cos a = \frac{4}{5}; \sin b = \frac{12}{13} \text{ and } 0 \leq b \leq 2 \Rightarrow \cos b = \frac{5}{13}}$$

$$\underline{\sin(a + b) = \sin a \cos b + \sin b \cos a = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}}$$

b)  $\cos(a + b)$

$$\underline{\cos(a + b) = \cos a \cos b - \sin a \sin b = \frac{20}{65} - \frac{36}{65} = -\frac{16}{65}}$$

c)  $\tan(a + b)$

$$\underline{\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \cdot \frac{12}{5}} = \frac{\frac{63}{20}}{1 - \frac{36}{20}} = \frac{-63}{16}}$$

d)  $\sin 2a$

$$\underline{\sin 2a = 2 \sin a \cos a = \frac{24}{25}}$$

e)  $\cos 2a$

$$\underline{\cos 2a = \cos^2 a - \sin^2 a = \frac{7}{25}}$$

**5.** Prove the following identities.

a)  $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \tan x}{\sec^2 x} = 2 \tan x \cdot \cos^2 x = \frac{2 \sin x}{\cos x} \cdot \cos^2 x = 2 \sin x \cos x = \sin 2x$

b)  $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \tan^2 x}{\sec^2 x} = (1 - \tan^2 x) \cdot \cos^2 x = \cos^2 x - \sin^2 x = \cos 2x$

c)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \tan x}{1 - \tan^2 x} = \frac{\frac{2 \sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{2 \sin x}{\cos x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x} \cdot \frac{\sin 2x}{\cos 2x} = \tan 2x$

**6.** Show that  $\tan 75^\circ$  has the exact value:  $2 + \sqrt{3}$ .

$$\underline{\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{(3 + \sqrt{3})^2}{(3 - \sqrt{3})(3 + \sqrt{3})} = \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}}$$