Identities Practice 1

Verify the following exercises; $u, A, x \in R$.

1. $\dfrac{\tan u}{\sec u} \equiv \sin u$

2. $\csc u \equiv \cot u \sec u$

3. $\sec u - \cos u \equiv \tan u \sin u$

4. $\cos^2 u - \sin^2 u \equiv 2\cos^2 u - 1$

5. $(\sec^2 u - 1)(\csc^2 u - 1) \equiv 1$

6. $\tan^2 u + \sec^2 u \equiv 2\sec^2 u - 1$

7. $(\tan u + \cot u)(\sin u \cos u) \equiv 1$

8. $\dfrac{\cos u - \sin u}{\cos u} \equiv 1 - \tan u$

9. $\dfrac{1}{1+\sin u} + \dfrac{1}{1-\sin u} \equiv 2\sec^2 u$

10. $\dfrac{1+\sec u}{\sec u} \equiv \dfrac{\sin^2 u}{1-\cos u}$

11. $\dfrac{1-\cos u}{\sin u} \equiv \dfrac{\sin u}{1+\cos u}$

12. $\dfrac{\sec u - 1}{\sec u + 1} \equiv \dfrac{1-\cos u}{1+\cos u}$

13. $\dfrac{\cos^2 u}{1-\sin u} \equiv \dfrac{\cos u}{\sec u - \tan u}$

14. $\cos u \cot u \equiv \dfrac{1}{\sin u} - \sin u$

15. $\tan A + \dfrac{1}{\tan A} \equiv \dfrac{\sec A}{\sin A}$

16. $\dfrac{\sin A + \tan A}{1+\sec A} \equiv \sin A$

17. $1 + \dfrac{1}{\cos A} \equiv \dfrac{\tan^2 A}{\sec A - 1}$

18. $\sin x + \cos x \equiv \dfrac{1+\tan x}{\sec x}$

19. $\tan u \equiv \dfrac{1+\sin u - \cos^2 u}{\cos u (1+\sin u)}$

20. $\cot u \equiv \dfrac{2+\csc u}{\sec u} - 2\cos u$

21. $\dfrac{\tan A \sin A}{\tan A + \sin A} \equiv \dfrac{\tan A - \sin A}{\tan A \sin A}$

22. $\dfrac{\sec^4 u + \tan^4 u}{\sec^2 u \tan^2 u} \equiv \dfrac{\cos^4 u}{\sin^2 u} + 2$

23. $\dfrac{\cot A + \csc A}{\sin A - \cot A - \csc A} \equiv -\sec A$

24. $\cot^2 A \sec^2 A \equiv 1 + \cot^2 A$

25. $\dfrac{\cot A + \cot B}{\tan A + \tan B} + \dfrac{1 - \cot A \cot B}{1 - \tan A \tan B} \equiv 0$

26. $\dfrac{\sin A \cos A}{1-\cos A} \equiv \dfrac{\sin^2 A}{\tan A - \sin A}$

27. $\sin u \sec u \cot u \equiv 1$

28. $\cot^2 u - \cos^2 u \equiv \cos^2 u \cot^2 u$

29. $\dfrac{\cos u}{1+\sin u} + \dfrac{1+\sin u}{\cos u} \equiv 2\sec u$

30. $\dfrac{1-\tan^2 u}{1-\cot^2 u} \equiv 1 - \sec^2 u$

31. $(\sec^2 u)(1 - \cos^2 u) \equiv \tan^2 u$

32. $\dfrac{\sin^4 u - \cos^4 u}{1 - \cot^4 u} \equiv \sin^4 u$

33. $\dfrac{1 - 2\sin u - 3\sin^2 u}{\cos^2 u} \equiv \dfrac{1 - 3\sin u}{1 - \sin u}$

Identities Practice 1 (solutions)

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1. $\dfrac{\tan x}{\sec x} = \sin x$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \sin x$$

$$\frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1} = \sin x$$

$$\sin x = \sin x$$

2. $\csc x = \cot x \cdot \sec x$

$$\csc x = \frac{\cancel{\cos x}}{\sin x} \cdot \frac{1}{\cancel{\cos x}}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

3. $\sec x - \cos x = \tan x \cdot \sin x$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$

common denomin.

$$\frac{1 - \cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \cdot \sin x$$

$$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$

Identities I (Solutions)

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4. $\cos^2 x - \sin^2 x \equiv 2\cos^2 x - 1$

$\cos^2 x - (1 - \cos^2 x)$

$\cos^2 x - 1 + \cos^2 x$

$2\cos^2 x - 1 \equiv 2\cos^2 x - 1$ ✓

5. $(\sec^2 x - 1)(\csc^2 x - 1) \equiv 1$

$(\tan^2 x)(\cos^2 x)$

$(\tan^2 x)\left(\frac{1}{\tan^2 x}\right)$

$1 \equiv 1$ ✓

6. $\tan^2 x + \sec^2 x \equiv 2\sec^2 x - 1$

$\tan^2 x + (1 + \tan^2 x)$

$2\tan^2 x + 1 \equiv 2(1 + \tan^2 x) - 1$

$\equiv 2 + 2\tan^2 x - 1$

$2\tan^2 x + 1 \equiv 2\tan^2 x + 1$ ✓

7. $(\tan x + \cot x)(\sin x \cos x) \equiv 1$

$$\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)(\sin x \cos x)$$

$$\frac{\sin x}{\cos x} \cdot \sin x \cdot \cos x + \frac{\sin x \cos^2 x}{\sin x}$$

$$\sin^2 x + \cos^2 x$$

$$1 \equiv 1$$ ✓

Identities 1 (solutions)

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8. $\dfrac{\cos x - \sin x}{\cos x} = 1 - \tan x$

$\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}$

$1 - \tan x = 1 - \tan x$ ✓

9. $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} \equiv 2\sec^2 x$ → get common denominator

$\dfrac{1}{(1+\sin x)(1-\sin x)}(1-\sin x) + \dfrac{1}{(1-\sin x)(1+\sin x)}(1+\sin x)$

FOIL →

$\dfrac{1-\sin x}{1-\sin^2 x} + \dfrac{1+\sin x}{1-\sin^2 x}$

$\dfrac{1-\sin x + 1+\sin x}{1-\sin^2 x} = 2\left(\dfrac{1}{\cos^2 x}\right)$

$\dfrac{2}{\cos^2 x} = \dfrac{2}{\cos^2 x}$ ✓

Identities 1 (solutions)

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10. $\dfrac{1+\sec x}{\sec x} \equiv \dfrac{\sin^2 x}{1-\cos x}$ ~~FACTOR!~~ difference of squares

$\dfrac{1}{\sec x} + \dfrac{\sec x}{\sec x} \equiv \dfrac{1-\cos^2 x}{1-\cos x}$

$\cos x + 1 \equiv \dfrac{(1+\cos x)(1-\cos x)}{1-\cos x}$

$\cos x + 1 \equiv \cos x + 1$

11. $\dfrac{1-\cos x}{\sin x} \equiv \dfrac{\sin x}{1+\cos x}$

$\equiv \dfrac{\sin x}{1+\cos x} \cdot \left(\dfrac{1-\cos x}{1-\cos x}\right)$ multiply by 1

$\equiv \dfrac{(\sin x)(1-\cos x)}{1-\cos^2 x}$ pythagorean identity

$\equiv \dfrac{(\sin x)(1-\cos x)}{\sin^2 x}$

$\dfrac{1-\cos x}{\sin x} \equiv \dfrac{1-\cos x}{\sin x}$ ✓