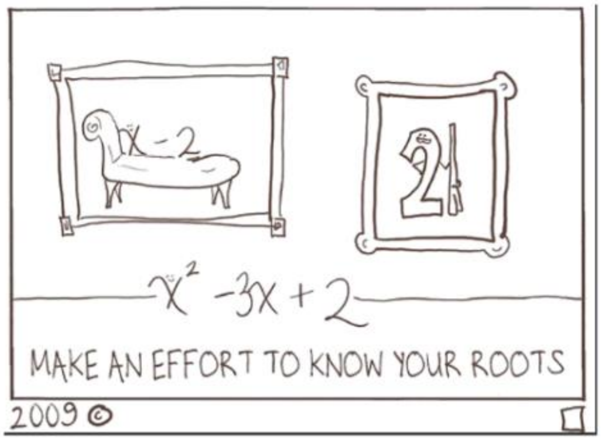
Lesson 3 Quadratic Formula

Date:

Chapter 3 Quadratic Equations

Lesson 3: Solving Equations: *Quadratic Formula*



Solving Quadratic Equations:

finding values for x that make the equation true

1. Factorization or Zero Product Principle
2. Isolate the variable square root b/s
3. Quadratic Formula

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solution, x, to an equation in the form:

$$ax^2 + bx + c = 0$$

ex $3x^2 - 7x + 10 = 0$

derrivation of QF

This is the original equation.	$ax^2 + bx + c = 0$
Move the loose number to the other side.	$ax^2 + bx = -c$
Divide through by whatever is multiplied on the squared term. Take half of the x-term, and square it. Add the squared term to both sides.	$x^2 + \frac{b}{a}x = -\frac{c}{a}$ $\frac{b}{2a} \to \frac{b^2}{4a^2}$ $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$
Simplify on the right-hand side; in this case, simplify by converting to a common denominator.	$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$
Convert the left-hand side to square form (and do a bit more simplifying on the right).	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
Square-root both sides, remembering to put the "±" on the right.	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$
Solve for "x =", and simplify as necessary.	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To use the **Quadratic Formula**

- quadratic equation in general form = 0
- there are no brackets on the LHS
- identify "a" "b" and "c"

$$1x^2 + 7x = -12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$1x^2 + 7x + 12 = 0$

1. $a = 1$
$b = 7$
$c = 12$

2. $\Delta = b^2 - 4ac$
$= (7)^2 - (4)(1)(12)$
$= 49 - 48$
$= 1$

3. $x = \frac{-7 \pm \sqrt{1}}{2(1)} = \frac{-7 \pm 1}{2}$

$x_1 = \frac{-7+1}{2} = -3$

$x_2 = \frac{-7-1}{2} = -4$

<u>discriminant</u>: tells you how many solutions (roots) to a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\Delta = b^2 - 4ac}$$

$\Delta > 0$ → 2 real roots

$\Delta < 0$ → <u>NO</u> real roots

$\Delta = 0$ → 1 real root

example

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 $3x^2 - 11x - 4 = 0$

$a = 3$

$b = -11$

$c = -4$

2 $\Delta = (-11)^2 - 4(3)(-4)$

$= 121 + 48$

$= 169$

$\Delta +$ ∴ 2 solutions

3 $x = \frac{-(-11) \pm \sqrt{169}}{2(3)}$

$= \frac{11 \pm 13}{6}$

$x_1 = \frac{11+13}{6} = \frac{24}{6}$

$x_2 = \frac{11-13}{6} = \frac{-2}{6}$

$\boxed{x = \{-\tfrac{1}{3}, 4\}}$

example

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1 $x^2 - 6x + 9 = 0$

$a = 1$

$b = -6$

$c = 9$

2 $\Delta = (-6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$ ∴ one solution

3 $x = \frac{-(-6) \pm \sqrt{0}}{2(1)}$

$= \frac{6}{2} = 3$

$\boxed{x = \{3\}}$

example

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 - x + 1 = 0$$

$a = 1$
$b = -1$
$c = 1$

$\Delta = (-1)^2 - 4(1)(1)$
$= 1 - 4$
$= -3$ $\Delta -$

No solution

$\boxed{x = \{\ \}}$ or $\boxed{x = \emptyset}$

example

$$2x^2 = x^2 + 4x + 37$$

$2x^2 - x^2 - 4x + 37$

$x^2 - 4x - 37 = 0$

1 $a = 1$
$b = -4$
$c = -37$

2 $\Delta = (-4)^2 - 4(1)(-37)$
$= 16 + 164$
$= 164$ $\Delta +$ $\therefore$ 2 sol'ns

3 $x = \frac{-(-4) \pm \sqrt{164}}{2(1)}$

$= \frac{4 \pm \sqrt{164}}{2}$

$x_1 = \frac{4 + \sqrt{164}}{2} \approx 8.403$

$x_2 = \frac{4 - \sqrt{164}}{2} \approx -4.403$

$\boxed{x = \{-4.403, 8.403}$

example

$$x^2 = -7x$$

$x^2 + 7x = 0$

1

$a = 1$

$b = 7$

$c = 0$

2

$\Delta = (7)^2 - (4)(1)(0)$

$= 49$

$\Delta +$

$\therefore$ 2 solutions

3

$x = \dfrac{-7 \pm \sqrt{49}}{2(1)}$

$= \dfrac{-7 \pm 7}{2}$

$x_1 = \dfrac{-7+7}{2}$
$= 0$

$x_2 = \dfrac{-7-7}{2}$
$= 7$

$\boxed{x = \{0, 7\}}$

example

$$-32x = 21 - 5x^2$$

$5x^2 - 32x - 21 = 0$

1

$a = 5$

$b = -32$

$c = -21$

2

$\Delta = (-32)^2 - 4(5)(-21)$

$= 1624 + 420$

$= 1444$

$\Delta +$

2 solutions

3

$x = \dfrac{-(-32) \pm \sqrt{1444}}{2(5)}$

$= \dfrac{32 \pm 38}{10}$

$x_1 = \dfrac{70}{10} = 7$

$x_2 = \dfrac{-6}{10} = -\dfrac{3}{5}$

$\boxed{x = \left\{-\dfrac{3}{5}, 7\right\}}$

example of a quadratic equation in a word problem

The value, in cents, of a share is given by $V = 2t^2 - 16t + 40$

where t represents the number of weeks since the share was purchased.

a) After how many weeks is the share worth 58 cents?

$\therefore$ find t, when $V = 58$

b) Will the value of the share ever drop to 6 cents?

$\therefore$ find t, when $V = 6$

a)

$$2t^2 - 16t + 40 = 58$$
$$2t^2 - 16t - 18 = 0$$
$$t^2 - 8t - 9 = 0$$

use Q.F. or Z.P.P.

$$(t-9)(t+1) = 0$$

$t - 9 = 0$ | $t + 1 = 0$

$t = 9$ | $\cancel{t = -1}$

weeks cannot be negative

$\boxed{\text{after 9 weeks}}$

b)

$$2t^2 - 16t + 40 = 6$$
$$2t^2 - 16t + 34 = 0$$
$$t^2 - 8t + 17 = 0$$

$a = 1$
$b = -8$
$c = 17$

$$\Delta = (-8)^2 - 4(1)(17)$$
$$= 64 - 68$$
$$= -4$$

Δ is neg

$\therefore$ no solution

$\boxed{\text{it can never drop to 6 cents}}$

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9. Using the sign of the discriminant, indicate the number of solutions to the following equations.

a) $2x^2 + 3x - 2 = 0$ ______ b) $-2x^2 - 5x + 3 = 0$ ______

c) $4x^2 + 12x + 9 = 0$ ______ d) $-x^2 + x - 1 = 0$ ______

e) $x^2 - 6x = 0$ ______ f) $2x^2 - 8 = 0$ ______

g) $x(x - 3) = -2$ ______ h) $-x^2 + 2x - 1 = 0$ ______

10. Solve the equations of the preceding exercise using the discriminant method.

a) S = $\boxed{-2,\ \tfrac{1}{2}}$ b) S = $\boxed{\tfrac{1}{2},\ -3}$ c) S = $\boxed{-\tfrac{3}{2}}$ d) S = $\boxed{\varnothing}$

e) S = $\boxed{0,\ 6}$ f) S = $\boxed{-2,\ 2}$ g) S = $\boxed{1,\ 2}$ h) S = $\boxed{1}$

a) $\Delta = (3)^2 - 4(2)(-2) = 25$ (2)

b) $\Delta = (5)^2 - 4(-2)(3) = 49$ (2)

c) $\Delta = (12)^2 - 4(4)(9) = 0$ (1)

d) $\Delta = (1)^2 - 4(-1)(-1) = -3$ $\varnothing$

e) $\Delta = (6)^2 - 4(1)(0) = 36$ (2)

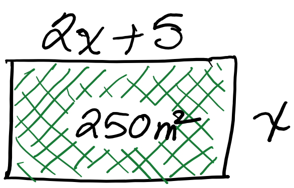
f) $\Delta = (0)^2 - 4(2)(-8) = 64$ (2)

g) $\Delta = (-3)^2 - 4(1)(2) = 1$ (2)

h) $\Delta = (2)^2 - 4(-1)(-1) = 0$ (1)

workbook page 34 #14

The length of a rectangular field measures 5 m more than twice its width. If the total area is equal to 250 m², what is the perimeter of the field?



(1) $(x)(2x+5) = 250$

$2x^2 + 5x - 250 = 0$

(2) Q.F.

$a = 2$

$b = 5$

$c = -250$

$\Delta = b^2 - 4ac$

(3) $= (5)^2 - 4(2)(-250)$

$= 25 + 2000$

$= 2025$

$\Delta +$ 2 solutions

(4) $x = \dfrac{-5 \pm \sqrt{2025}}{2(2)}$

$x = \dfrac{-5 \pm 45}{4}$

$x_1 = \dfrac{-5+45}{4} = 10$

$x_2 = \dfrac{-5-45}{4} = \cancel{-12.5}$ reject b/c dimensions cannot be negative

$\boxed{\therefore\ P = (10 + 25) \times 2 = 70 \text{ m}}$

workbook Page 34 # 17

A mother is presently 5 years older than twice her daughter's age. Ten years ago, the product of the mother and daughter's ages was equal to 125. What is the mother's present age?

10 years ago

	now	then
mom	$2x+5$	$2x+5-10$
daughter	x	$x-10$

① $(2x-5)(x-10)=125$

$2x^2-20x-5x+50=125$

$2x^2-25x-75=0$

② Q.F.

$a=2$

$b=-25$

$c=-75$

③ $\Delta=(-25)^2-4(2)(-75)$

$=1225$ $\Delta +$ 2 solutions

④ $x=\dfrac{-(-25)\pm\sqrt{1225}}{2(2)}$

$x_1=15$ $x_2=-2.5$ reject a negative age

$\therefore$ $\boxed{\text{mom is } 35}$

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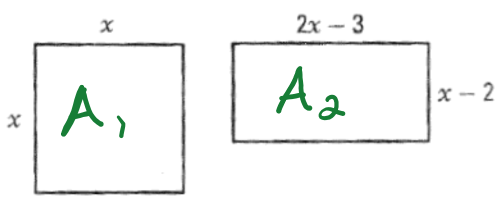
Square and rectangle have equal area. What is the actual perimeter of the rectangle?



$A_1=A_2$

$x^2=(2x-3)(x-2)$

$x^2=2x^2-4x-3x+6$

$0=x^2-7x+6$

Z.P.P. $0=(x-6)(x-1)$

$x-6=0$ | $x-1=0$

$x=6$ | $x=1$

reject as it will give a negative dimension

$2(6)-3=9$

$6-2=4$

$P=9+4+9+4$

$\therefore$ $\boxed{P=26u}$

you can now do;

Worksheets

- Solving Equations Using Quadratic Formula
- Solving Equations Using Quadratic Formula 2

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