

14) $\sec x - \cos x = \sin x \tan x$

a) $\frac{1}{\cos} - \cos x = \sin x \tan x$

$\frac{1}{\cos} - \frac{\cos^2 x}{\cos x} = \sin x \tan x$

$\frac{1 - \cos^2 x}{\cos x} = \sin x \tan x$

$\frac{\sin^2 x}{\cos x} = \sin x \frac{\sin x}{\cos x}$

$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} \quad \checkmark$

bring these 2 together \therefore get a common denominator.

b) $\frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$

$\frac{(1 + \sin x)(1 - \sin x)}{(\cos x)(1 - \sin x)} = \frac{\cos x}{(1 - \sin x)}$

started by looking for a common denominator

$\frac{1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x}{(\cos x)(1 - \sin x)} = "$

$\frac{1 - \sin^2 x}{(\cos x)(1 - \sin x)} = "$

$\frac{\cos^2 x}{(\cos x)(1 - \sin x)} = "$

$\frac{\cos x}{1 - \sin x}$

In the end, I did not need one!

$$c) \frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$$

$$\frac{1 - \cos^2 x}{1 - \cos x} = 1 + \cos x$$

$$\frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x} = 1 + \cos x$$

$$1 + \cos x = 1 + \cos x$$

I see there is
a difference of
squares here

$$d) (1 + \tan x)^2 + (1 - \tan x)^2 = 2 \sec^2 x$$

$$\underbrace{(1 + \tan x)(1 + \tan x)}_{\text{FOIL}} + \underbrace{(1 - \tan x)(1 - \tan x)}_{\text{FOIL}} = 2 \sec^2 x$$

$$\textcircled{1} + \cancel{\tan x} + \cancel{\tan x} + \tan^2 x + \textcircled{1} + \cancel{\tan x} - \cancel{\tan x} + \tan^2 x = 2 \sec^2 x$$

$$2 + 2 \tan^2 x = 2 \sec^2 x$$

$$2 + 2 \tan^2 x = 2(1 + \tan^2 x) \quad \text{Pythagorean identity}$$

$$2 + 2 \tan^2 x = 2 + 2 \tan^2 x$$