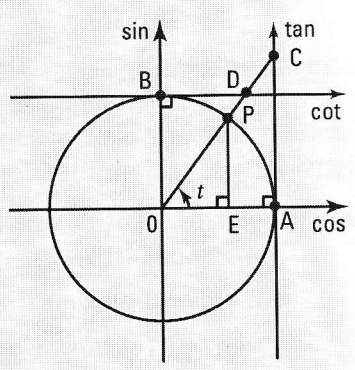
BASIC TRIGONOMETRIC IDENTITIES

- If P(t) is a trigonometric point, then
  $m\overline{0E} = \cos t$, $m\overline{PE} = \sin t$.
  $m\overline{AC} = \tan t$, $m\overline{BD} = \cot t$.
  $m\overline{0C} = \sec t$, $m\overline{0D} = \csc t$.
- 1st basic identity:

$$\sin^2 t + \cos^2 t = 1$$

- Other basic identities:

$$1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t$$



1. Verify the three basic identities when $t = \frac{\pi}{6}$.

$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6} = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$

$1 + \tan^2 \frac{\pi}{6} = 1 + \left(\frac{\sqrt{3}}{3}\right)^2 = 1 + \frac{1}{3} = \frac{4}{3}$; $\sec^2 \frac{\pi}{6} = \frac{1}{\cos^2 \frac{\pi}{6}} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

$1 + \cot^2 \frac{\pi}{6} = 1 + \left(\frac{3}{\sqrt{3}}\right)^2 = 1 + 3 = 4$; $\csc^2 \frac{\pi}{6} = \frac{1}{\sin^2 \frac{\pi}{6}} = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$.

2. Using an angle measure t of your choice, expressed in radians or in degrees, verify the three basic identities.

Various answers.

3. Use the appropriate basic identity to calculate

a) $\sin t$, knowing that $\cos t = \frac{3}{5}$ and $270° \leq t \leq 360°$.

$\sin^2 t = 1 - \cos^2 t = \frac{16}{25} \Rightarrow \sin t = \frac{-4}{5}$

b) $\cos t$, knowing that $\sin t = \frac{40}{41}$ and $90° \leq t \leq 180°$.

$\cos^2 t = 1 - \sin^2 t = \frac{81}{1600} \Rightarrow \cos t = \frac{-9}{40}$

c) $\tan t$, knowing that $\sec t = \frac{-5}{4}$ and $180° \leq t \leq 270°$.

$\tan^2 t = \sec^2 t - 1 = \frac{9}{16} \Rightarrow \tan t = \frac{3}{4}$

d) $\cot t$, knowing that $\csc t = -\frac{13}{12}$ and $270° \leq t \leq 360°$.

$\cot^2 t = \csc^2 t - 1 = \frac{25}{144} \Rightarrow \cot t = \frac{-5}{12}$

e) $\sec t$, knowing that $\tan t = \frac{7}{24}$ and $180° \leq t \leq 270°$.

$\sec^2 t = 1 + \tan^2 t = \frac{625}{576} \Rightarrow \sec t = \frac{-25}{24}$

f) $\csc t$, knowing that $\cot t = \frac{4}{3}$ and $0° \leq t \leq 90°$.

$\csc^2 t = 1 + \cot^2 t = \frac{25}{9} \Rightarrow \csc t = \frac{5}{3}$