ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

- The denominators have no common factor.

$$\frac{A(x)}{B(x)} \pm \frac{C(x)}{D(x)} = \frac{A(x)\,D(x) \pm B(x)\,C(x)}{B(x)\,D(x)}$$

Ex.: $\frac{2}{x-1} + \frac{3}{x+3} = \frac{2(x+3)+3(x-1)}{(x-1)(x+3)}$

$= \frac{2x+6+3x-3}{(x-1)(x+3)}$

$= \frac{5x+3}{(x-1)(x+3)}$

Here, the product of the denominators is the common denominator.

Ex.: $\frac{2x}{x+1} - \frac{3x}{x-1} = \frac{2x(x-1)-3x(x+1)}{(x+1)(x-1)}$

$= \frac{2x^2-2x-3x^2-3x}{(x+1)(x-1)}$

$= \frac{-x^2-5x}{(x+1)(x-1)}$

- The denominators have a common factor.

Ex.: $\frac{4}{x^2-9} - \frac{2}{(x+3)^2} = \frac{4}{(x+3)(x-3)} - \frac{2}{(x+3)^2}$ Factor the denominators.

$= \frac{4(x+3)}{(x+3)^2(x-3)} - \frac{2(x-3)}{(x+3)^2(x-3)}$ Find the common denominator using the least number of factors.

$= \frac{4(x+3)-2(x-3)}{(x+3)^2(x-3)}$ Reduce to a common denominator.

$= \frac{2x+18}{(x+3)^2(x-3)}$ Simplify.

$(x+3)^2(x-3)$ is the lowest common denominator.

In each of the following operations, the variable does not take values which make the denominators zero.

5. Perform the following operations.

a) $\frac{2}{x} + \frac{5}{y}$ ***2y + 5x / xy***

b) $\frac{3}{x} - \frac{2}{x^2}$ ***3x − 2 / x²***

c) $\frac{x+y}{x} - \frac{x+y}{y}$ ***y² − x² / xy***

d) $\frac{2x+3y}{9x} + \frac{2x-3y}{6y}$ ***6x²+ 6y² − 5xy / 18xy***

6. Perform the following operations.

a) $\frac{2}{x+1} + \frac{3}{x-2}$ ***5x − 1 / (x + 1)(x − 2)***

b) $\frac{3}{x+2} - \frac{2}{x-2}$ ***x − 10 / x² − 4***

c) $\frac{3x}{x-1} + \frac{2x}{x+1}$ ***5x² + x / x² − 1***

d) $\frac{x-2}{x+3} - \frac{x-3}{x+2}$ ***5 / (x + 3)(x + 2)***

7. Perform the following operations and simplify your answer.

a) $\frac{x}{x^2-9} - \frac{1}{2x-6}$ ***1 / 2x + 6***

b) $\frac{1}{x+5} + \frac{1}{x-5}$ ***2x / x² − 25***

c) $\frac{1}{x^2-2x+1} + \frac{1}{x-1}$ ***x / (x − 1)²***

d) $\frac{2a}{3a-15} + \frac{4a}{2a-10}$ ***8a / 3a − 15***

8. Perform the following additions and subtractions.

a) $\frac{2x+1}{2} - \frac{3x-1}{5}$ ***4x + 7 / 10***

b) $\frac{5}{x-2} + \frac{3}{x+3}$ ***8x + 9 / (x − 2)(x + 3)***

c) $\frac{x+1}{x-2} - \frac{x+2}{x-1}$ ***3 / (x − 2)(x − 1)***

d) $\frac{x+3}{x-3} - \frac{x-3}{x+3}$ ***12x / x² − 9***

e) $\frac{x+1}{x^2-4} - \frac{x-2}{x+2}$ ***−x² + 5x − 3 / x² − 4***

f) $\frac{x+1}{x^2-2x+1} + \frac{1}{x^2-1}$ ***x² + 3x / (x − 1)²(x + 1)***