

8. Solve the following equations using the formulas stated in the theory box on page 32.

- a) $(x - 1)^2 + 9 = 0$ $S = \emptyset$ b) $2(x - 3)^2 = 0$ $S = \{3\}$
 c) $(2x - 5)^2 - 25 = 0$ $S = \{0, 5\}$ d) $2(x + 1)^2 - 14 = 0$ $S = \{-1 - \sqrt{7}, -1 + \sqrt{7}\}$
 e) $-2(2x + 1)^2 + 32 = 0$ $S = \{-\frac{5}{2}, \frac{3}{2}\}$ f) $3(x - 2)^2 + 27 = 0$ $S = \emptyset$

SOLVING QUADRATIC EQUATIONS: THE DISCRIMINANT METHOD

- The discriminant, noted Δ (read delta), of the quadratic equation $ax^2 + bx + c = 0$ is the real number:

$$\Delta = b^2 - 4ac$$

- The existence and number of real solutions depend on the sign of the discriminant Δ .

Sign of Δ	Number of solutions	Solutions
$\Delta > 0$	2 solutions	$x_1 = \frac{-b - \sqrt{\Delta}}{2a}$ and $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$
$\Delta = 0$	1 solution	$x = -\frac{b}{2a}$
$\Delta < 0$	no real solution	

$3x^2 - 11x - 4 = 0$

- $a = 3, b = -11, c = -4$
- $\Delta = b^2 - 4ac$
- $\Delta = (-11)^2 - 4 \times 3 \times -4$
- $\Delta = 169$
- $x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{11 - 13}{6} = -\frac{1}{3}$
- $x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{11 + 13}{6} = 4$

Thus, $S = \{-\frac{1}{3}, 4\}$.

$x^2 - 6x + 9 = 0$

- $a = 1, b = -6, c = 9$
- $\Delta = b^2 - 4ac$
- $\Delta = (-6)^2 - 4 \times 1 \times 9$
- $\Delta = 0$
- $x = \frac{-b}{2a} = \frac{6}{2} = 3$

Thus, $S = \{3\}$.

$x^2 - x + 1 = 0$

- $a = 1, b = -1, c = 1$
- $\Delta = b^2 - 4ac$
- $\Delta = (-1)^2 - 4 \times 1 \times 1$
- $\Delta = -3$

Thus, $S = \emptyset$.

9. Using the sign of the discriminant, indicate the number of solutions to the following equations.

- a) $2x^2 + 3x - 2 = 0$ $\Delta = 25; 2 \text{ solutions}$ b) $-2x^2 - 5x + 3 = 0$ $\Delta = 49, 2 \text{ solutions}$
 c) $4x^2 + 12x + 9 = 0$ $\Delta = 0; 1 \text{ solution}$ d) $-x^2 + x - 1 = 0$ $\Delta = -3; 0 \text{ solution}$
 e) $x^2 - 6x = 0$ $\Delta = 36; 2 \text{ solutions}$ f) $2x^2 - 8 = 0$ $\Delta = 64; 2 \text{ solutions}$
 g) $x(x - 3) = -2$ $\Delta = 1; 2 \text{ solutions}$ h) $-x^2 + 2x - 1 = 0$ $\Delta = 0; 1 \text{ solution}$

10. Solve the equations of the preceding exercise using the discriminant method.

- a) $S = \{-2, \frac{1}{2}\}$ b) $S = \{-3, \frac{1}{2}\}$ c) $S = \{-\frac{3}{2}\}$ d) $S = \emptyset$
 e) $S = \{0, 6\}$ f) $S = \{-2, 2\}$ g) $S = \{1, 2\}$ h) $S = \{1\}$