7. The following functions have the rule $f(x) = a \cos b(x - h) + k$.
Find the zeros of each function over
1. the interval $[h, h + p]$ where p is the period of the function.
2. the set of all real numbers.

a) $f(x) = 2\cos\frac{\pi}{6}(x - 2) + 1$

$\cos\frac{\pi}{6}(x - 2) = -\frac{1}{2}$

$\frac{\pi}{6}(x - 2) = \frac{2\pi}{3}$ or $\frac{\pi}{6}(x - 2) = \frac{4\pi}{3}$

$x = 6$ or $x = 10$

1. $S = \{6, 10\}$
2. $S = \{6 + 12n\} \cup \{10 + 12n\}$

b) $f(x) = -2\cos\frac{\pi}{3}(x + 1) + \frac{1}{2}$

$\cos\frac{\pi}{3}(x + 1) = \frac{1}{4}$

$\frac{\pi}{3}(x + 1) = 1.32$ or $\frac{\pi}{3}(x + 1) = 4.97$

$x = 0.26$ or $x = 3.75$

1. $S = \{0.26; 3.75\}$
2. $S = \{0.26 + 6n\} \cup \{3.75 + 6n\}$

c) $f(x) = \cos 2(x - \pi) + 1$

$\cos 2(x - \pi) = -1$

$2(x - \pi) = \pi$

$x = \frac{3\pi}{2}$

1. $S = \left\{\frac{3\pi}{2}\right\}$
2. $S = \left\{\frac{3\pi}{2} + \pi n\right\}$

d) $f(x) = 2\cos\left(x + \frac{\pi}{2}\right) - \sqrt{3}$

$\cos\left(x + \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$

$x + \frac{\pi}{2} = \frac{\pi}{6}$ or $x + \frac{\pi}{2} = \frac{11\pi}{6}$

$x = -\frac{\pi}{3}$ or $x = \frac{4\pi}{3}$

1. $S = \left\{\frac{-\pi}{3}, \frac{4\pi}{3}\right\}$
2. $S = \left\{\frac{-\pi}{3} + 2\pi n\right\} \cup \left\{\frac{4\pi}{3} + 2\pi n\right\}$

e) $f(x) = -3\cos\frac{\pi}{4}x + 6$

$\cos\frac{\pi}{4}x = 2$

1. $S = \varnothing$
2. $S = \varnothing$

f) $f(x) = -2\cos\frac{\pi}{8}(x + 2) - \sqrt{2}$

$\frac{\pi}{8}(x + 2) = \frac{3\pi}{4}$ or $\frac{\pi}{8}($

$\frac{\pi}{8}(x + 2) = \frac{3\pi}{4}$ or $\frac{\pi}{8}(x + 2) = \frac{5\pi}{4}$

$x = 4$ $x = 8$

1. $S = \{4, 8\}$
2. $S = \{4 + 16n\} \cup \{8 + 16n\}$

8. Determine the zeros of the function $f(x) = -2\cos\frac{\pi}{12}(x + 5) - 1$ over the interval [66, 12

$\cos\frac{\pi}{12}(x + 5) = -\frac{1}{2}$

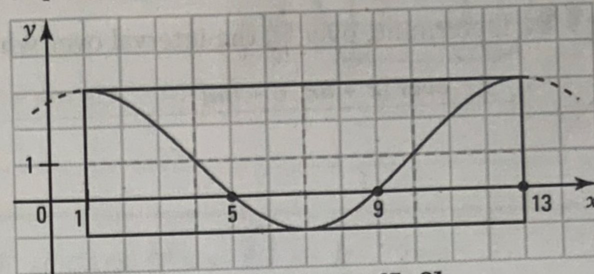
$\frac{\pi}{12}(x + 5) = \frac{2\pi}{3}$ or $\frac{\pi}{12}(x + 5) = \frac{4\pi}{3}$

$x = 3$ or $x = 11$

The zeros over the interval [66, 126] are 75, 83, 99, 107, 123.

ACTIVITY 5 Study of the function $f(x) = a \cos b(x - h) + k$

One cycle of the function $f(x) = 2\cos\frac{\pi}{6}(x-1)+1$ is represented below.



a) Determine

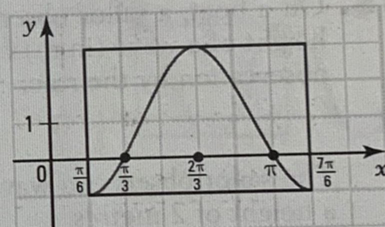
1. the period. ***12***
2. the amplitude. ***2***

b) Determine

1. dom *f*. ***ℝ***
2. ran *f*. ***[−1, 3]***
3. the zeros of *f* over [1, 13]. ***5 and 9***
4. the zeros of *f* over ℝ. ***{5 + 12n} ∪ {9 + 12n}***
5. the sign of *f* over [1, 13]. ***f(x) ≥ 0 over [1, 5] ∪ [9, 13] and f(x) ≤ 0 over [5, 9]***
6. the sign of *f* over ℝ. ***f(x) ≥ 0 over [1 + 12n, 5 + 12n] ∪ [9 + 12n, 13 + 12n]***
   f(x) ≤ 0 over [5 + 12n, 9 + 12n]
7. the variation of *f* over [1, 13]. ***f ↘ 0 over [1, 7] and f ↗ 0 over [7, 13]***
8. the variation of *f* over ℝ. ***f ↘ 0 over [1 + 12n, 7 + 12n] and f ↗ 0 over [7 + 12n, 13 + 12n]***
9. the maximum and minimum of *f*. ***max f = 3; min f = −1***

STUDY OF THE FUNCTION $f(x) = a \cos b(x - h) + k$

Given $f(x) = -2\cos 2\left(x - \frac{\pi}{6}\right) + 1$. We have:



- dom $f = \mathbb{R}$; ran $f = [-1, 3]$
- period $p = \pi$; amplitude $A = 2$
- zeros of f: $\left\{\frac{\pi}{3} + \pi n\right\} \cup \{\pi + \pi n\}$
- sign of f.

  $f(x) \leq 0$ over $\left[\frac{\pi}{6} + \pi n, \frac{\pi}{3} + \pi n\right] \cup \left[\pi + \pi n, \frac{7\pi}{6} + \pi n\right]$

  $f(x) \geq 0$ over $\left[\frac{\pi}{3} + \pi n, \pi + \pi n\right]$
- variation of f.

  $f \nearrow$ over $\left[\frac{\pi}{6} + \pi n, \frac{2\pi}{3} + \pi n\right]$; $f \searrow$ over $\left[\frac{2\pi}{3} + \pi n, \frac{7\pi}{6} + \pi n\right]$
- $\min f = -1$; $\max f = 3$.

9. For each of the following functions, determine

1. the period,
2. the amplitude,
3. the range of the function.

a) $f(x) = 2\cos\frac{3\pi}{4}(x-1)+5$

1. ***$p = \frac{8}{3}$***
2. ***A = 2***
3. ***ran f = [3, 7]***

b) $f(x) = 5\cos 4(x+2) - 12$

1. ***$p = \frac{\pi}{2}$***
2. ***A = 5***
3. ***ran f = [−17, −7]***

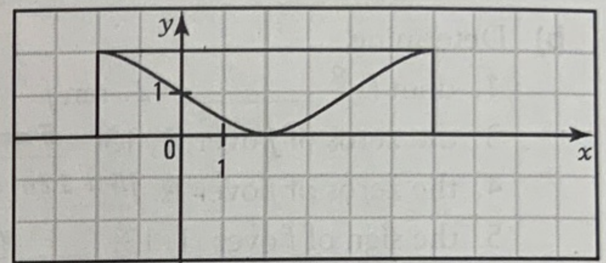
10. Determine the initial value of the following functions.

a) $f(x) = 2\cos\frac{\pi}{3}(x+1)+1$ ***2***

b) $f(x) = 5\cos 2\left(x-\frac{\pi}{2}\right)+1$ ***-4***

11. Determine, over $\mathbb{R}$, the interval over which the function $f(x) = \cos\frac{\pi}{4}(x+2)+1$ is increasing.

f ↗ over [2 + 8n, 6 + 8n]



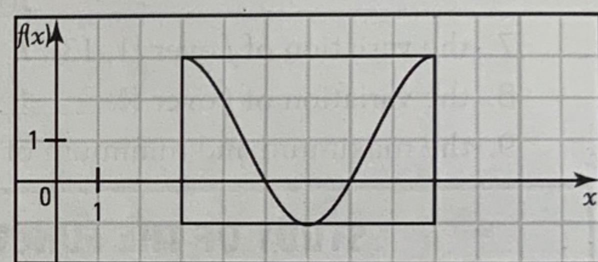
12. Determine, over $\mathbb{R}$, the interval over which the function $f(x) = 2\cos\frac{\pi}{3}(x-3)+1$ is negative.

Zeros: $\cos\frac{\pi}{3}(x-3) = -\frac{1}{2}$

$\frac{\pi}{3}(x-3) = \frac{2\pi}{3}$ **or** $\frac{\pi}{3}(x-3) = \frac{4\pi}{3}$

$x = 5$ **or** $x = 7$

$f(x) \leq 0$ ***over [5 + 6n, 7 + 6n]***



13. On a boat, a sailor observes the movement of the waves while on a sailing expedition. The height of a wave (in m) can be expressed as a function of the time (in s) since the start of the observation by the rule:

$$h(t) = 2\cos\frac{\pi}{6}(t-4)+1$$

The sailor observes a wave for 30 seconds. Determine at which moments the wave will be at a height of 2 metres.

$2\cos\frac{\pi}{6}(t-4)+1 = 2$

$\cos\frac{\pi}{6}(t-4) = \frac{1}{2}$

$\frac{\pi}{6}(t-4) = \frac{\pi}{3}$ **or** $\frac{\pi}{6}(t-4) = \frac{5\pi}{3}$

$t = 6$ **or** $t = 14$

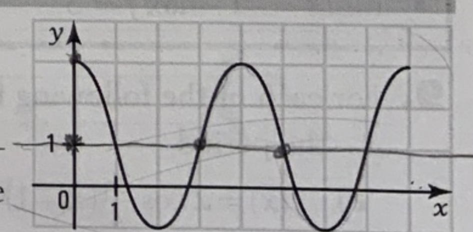
The wave will be at a height of 2 m at the moments t = 2 s, t = 6 s, t = 14 s, t = 18 s and t = 26 s.

ACTIVITY 6 Finding the rule $y = a\cos b(x-h)+k$

Consider the function f represented on the right.

a) Determine

1. the period of f. ***4***
2. the amplitude of f. ***2***

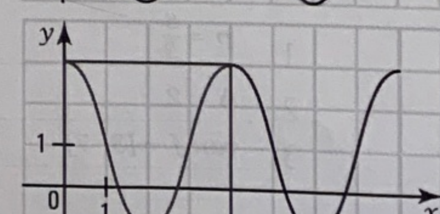


b) Determine the rule of the function f when we choose a cycle starting at the point $(h, k + A) = (0, 3)$.

Explain your procedure: ***Since the cycle is decreasing from the start, we deduce that a > 0. Thus, a = 2 and k = 1***

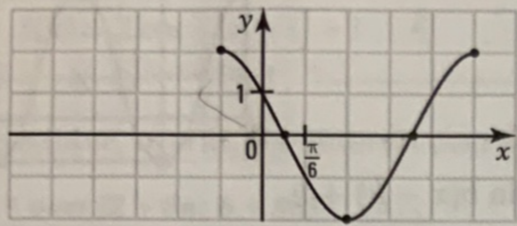
$p = 4 \Rightarrow |b| = \frac{\pi}{2}$.

Thus, $f(x) = 2\cos\frac{\pi}{2}x + 1$ ***or*** $f(x) = 2\cos\frac{-\pi}{2}x + 1$



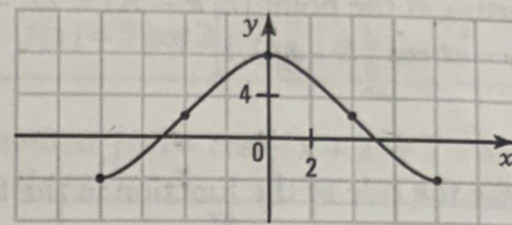
14. Find a rule of the form $y = a \cos b(x - h) + k$ for each of the following functions.

a)



For example, $y = 2 \cos 2\left(x + \frac{\pi}{6}\right)$

b)



For example, $y = -6 \cos \frac{\pi}{8}(x + 8) + 2$

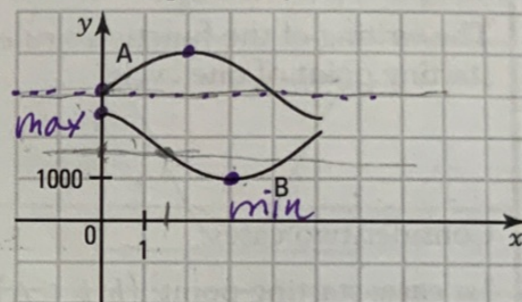
15. The populations of two neighboring villages A and B vary according to the model of a sinusoidal function which gives the population P of the village as a function of time t, in years, since the year 2000.

In the year 2000, the two villages had 3000 and 2500 inhabitants respectively.

The graph on the right shows the progression of the population of each village.

Village A reaches its maximum population of 4125 after 2 years and village B reaches its minimal population of 1000 after 3 years.

What will be the difference in population between these two villages in the year 2005?



Rule corresponding to village A.	Rule corresponding to village B.
$(h, k) = (0, 3000)$	$(h, k + A) = (0, 2500)$
$A = 1125$	$A = 750$
$p = 8 \Rightarrow b = \frac{\pi}{4}$	$p = 6 \Rightarrow b = \frac{\pi}{3}$
$y = 1125 \sin \frac{\pi}{4}x + 3000$	$y = 750 \cos \frac{\pi}{3}x + 1750$
In 2005, $y = 2205$ inhabitants	In 2005, $y = 2125$ inhabitants

The difference in their populations will be 80 inhabitants.

#15 You have to
assume that for
Village A → baseline
is 3000
and for Village B
2500 is a maximum

test questions will be
more clear