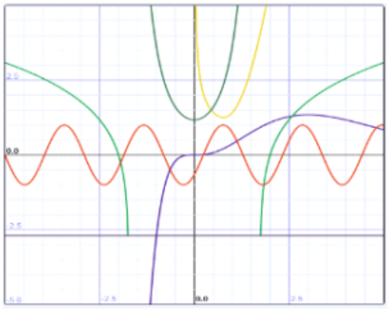
Lesson 3 Standard Form and General Form

Date:

Chapter 4: Linear and Quadratic Functions:



Lesson 3:

Standard and General Form

Graph the **basic** Quadratic Function

$$f(x) = x^2$$

eventually we will look at something like:

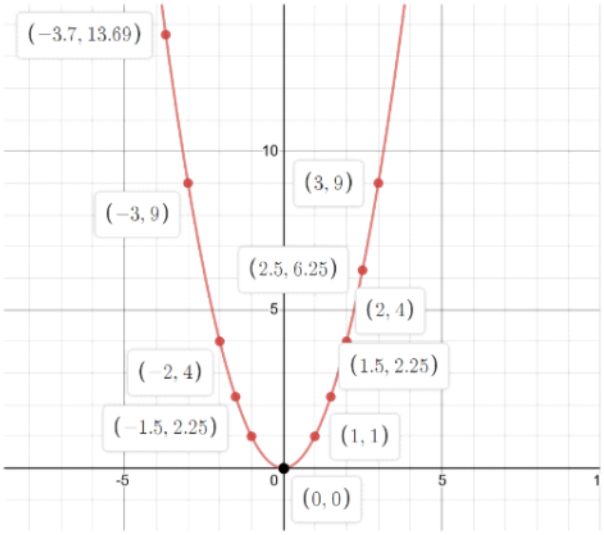
$f(x) = -0.25\,(x-6)^2 + 17.2$

$$f(x) = x^2$$

"when you have the RULE you have *everything*"

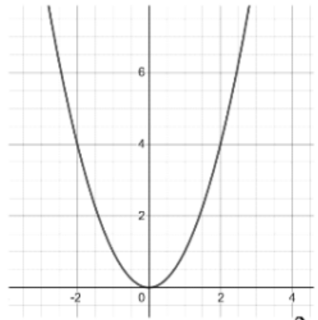
x	$f(x)$
-3.7	13.69
-3	9
-2	4
-1.5	2.25
0	0
1	1
1.5	2.25
2	4
2.5	6.25
3	9

$$f(x) = x^2$$



DESMOS

you have the App on your phone



$f(x) = x^2$

on desmos: $f(x) = ax^2$

$f(x) = 2x^2$

$f(x) = -3x^2$

$f(x) = 4x^2$

$f(x) = -.25x^2$

$f(x) = \frac{1}{2}x^2$

What do you notice?

a + "smiles"

a − "frowns"

- as a gets further from 0 → NARROW
- as a get closer to 0 → WIDE

on desmos: $f(x)=(x-h)^2$ $x - \boxed{}$

$f(x)=(x-2)^2$ $x - \boxed{+2}$ R

$f(x)=(x+1)^2$ $x - \boxed{-1}$ L

$f(x)=(x-6)^2$ $x - \boxed{+6}$ R.

$f(x)=(x-5.5)^2$ $x - \boxed{+5.5}$ R.

$f(x)=(x+4)^2$ $x - \boxed{-4}$ L

$f(x)=(x+7)^2$ $x - \boxed{-7}$ L

What do you notice?

- shifts **RIGHT** when **h** is positive
- shifts **LEFT** when **h** is negative

on desmos: $f(x)=(x)^2+k$

$f(x)=(x)^2+4$

$f(x)=(x)^2-6.2$

$f(x)=(x)^2+7$

$f(x)=(x)^2+8$

$f(x)=(x)^2-3$

$f(x)=(x)^2-1$

what do you notice?

- shifts **UP** when **k** is positive
- shifts **DOWN** when **k** is negative

Bring it all together

$$f(x) = a(x-h)^2 + k$$

a ?

width & direction of opening

a+   a−

h ?

horizontal shift

h +

h−

k ?

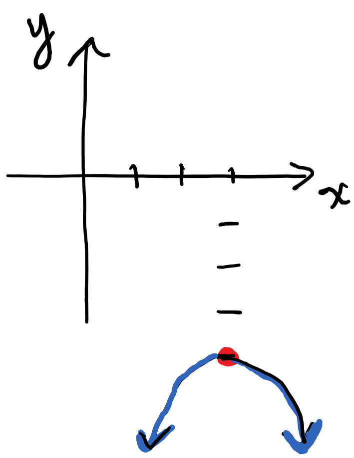
vertical shift

k+

k−

ex $f(x) = -2(x-3)^2 - 4$

in standard form we are quickly able to determine the vertex



if you are required to have an "accurate" sketch

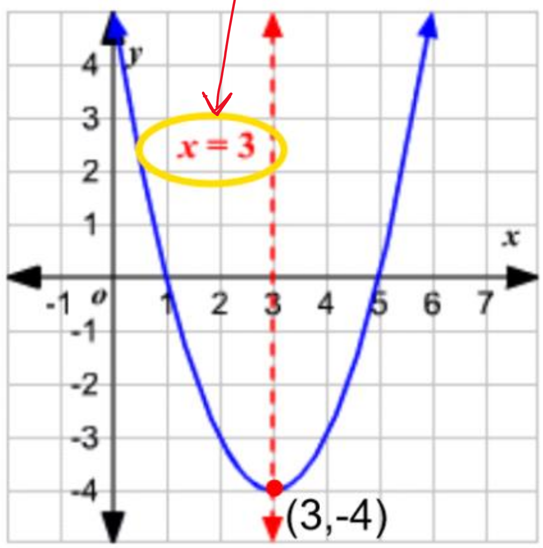
↳ table of values

We need the vertex to know **the equation of the axis of symmetry**

of the axis of symmetry

Vertical lines are ALWAYS written as

$\boxed{x = \#}$



Notes

Standard Form of a Quadratic Function

$$f(x) = a(x-h)^2 + k$$

a) $f(x) = 2(x+1)^2 - 3$

b) $-3(x-4)^2 + 2$

c) $.76(x-5)^2 + 10$

d) $4(x-4)^2 - 3$

e) $-5(x-6)^2 + 3$

f) $-\frac{3}{2}(x+2)^2 - 1$

g) $1.1(x+5)^2 - 3$

h) $-8(x-3)^2 + 4$

i) $-.75(x)^2 - 1$

for each...

- vertex?
- opening?
- width compared to basic?
- equation of axis of symmetry?

a) $(-1,3)$ N $x=-1$

b) $(4,2)$ N $x=4$

c) $(5,10)$ W $x=5$

d) $(4,-3)$ N $x=4$

e) $(6,3)$ N $x=6$

f) $(-2,-1)$ N $x=-2$

g) $(-5,-3)$ N $x=-5$

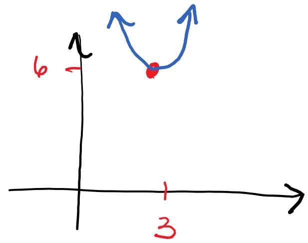
h) $(3,4)$ N $x=3$

i) $(0,1)$ W $x=0$

The vertex is what we need when we are trying to determine the range of a 2nd degree function.

ex. $f(x) = 2(x-3)^2 + 6$

DRAW a sketch



$\boxed{\text{range : } [6, \infty}$

Moving from *Standard Form* to *General Form*

$f(x) = a(x-h)^2 + k \qquad f(x) = ax^2 + bx + c$

ex. $f(x) = -2(x+5)^2 - 3$

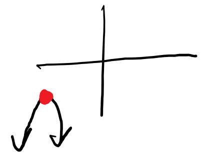
$$\left.\begin{aligned} &= -2(x+5)(x+5) - 3 \\ &= -2(x^2 + 10x + 25) - 3 \\ &= -2x^2 - 20x - 50 - 3 \\ &= -2x^2 - 20x - 53 \end{aligned}\right\} \text{you simply 'expand'!}$$

When you have the RULE......you have.....?

EVERYTHING

What is the **range** of the 2nd degree polynomial function?

$f(x) = -2(x+4.5)^2 - 2$



range: $-\infty, -2]$

What is the **y-intercept** of the 2nd degree polynomial function?

let $x = 0$

$f(0) = -2(0+4.5)^2 - 2$

$= -2(4.5)^2 - 2$

$= -2(20.25) - 2$

$= -40.5 - 2$

$\boxed{f(0) = -42.5}$

What is the range of the 2nd degree polynomial function?

$f(x) = -2x^2 - 28x - 101$

hmmmm ?

need the vertex

What is the y-intercept of the 2nd degree polynomial function?

$f(0) = -2(0)^2 - 28(0) - 101$

always!

To change from General to Standard Form

"complete the square"

Take "a" as a common factor from first 2 terms.

$f(x) = -2x^2 - 28x - 101$

$= -2(x^2 + 14x) - 101$

$= -2(x^2 + 14x \qquad) - 101$

$= -2(x^2 + 14x + 49 - 49) - 101$

$= -2(x^2 + 14x + 49) + 98 - 101$

$= -2(x+7)^2 - 3$

you need to "apply" the -2 to the -49 when you remove it from the bracket.

the vertex is: (-7,-3)

y-intercept is: -101

1st 3 terms : PST → can always be written as a $(\text{binomial})^2$

Another example:

$f(x) = 3x^2 + 6x - 6$

$= 3(x^2 + 2x \qquad) - 6$

$= 3(x^2 + 2x + 1 - 1) - 6$ $\times 3$

$= 3(x^2 + 2x + 1) - 3 - 6$

$= 3(x+1)^2 - 9$

Factor the "a" from the first 2 terms

PST – can always be written as a $(\text{binomial})^2$

Sometimes it is messy.. but it is still possible

$f(x) = -3x^2 + 5x + 19$

$= -3(x^2 - \frac{5}{3}x \qquad) + 19$

PST $= -3(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}) + 19$ $\quad x-3$

$= -3(x^2 - \frac{5}{3}x + \frac{25}{36}) + \frac{25}{12} + 19$

$= -3(x^2 - \frac{5}{3}x + \frac{25}{36}) + \frac{25}{12} + \frac{228}{12}$

$= -3(x^2 - \frac{5}{6})^2 + \frac{253}{12}$

- Factoring -3 from 5 simply creates a fraction
- To take half of any fraction, simply double the denominator

$\therefore \frac{1}{2}$ of $\frac{5}{3} = \frac{5}{6}$

One more

$f(x) = -2x^2 + 7x - 1$

$= -2(x^2 - \frac{7}{2}x \qquad) - 1$

PST $= -2(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}) - 1$ $\quad x-2$

$= -2(x^2 - \frac{7}{2}x + \frac{49}{16}) + \frac{49}{8} - 1$

$= -2(x - \frac{7}{2}x + \frac{49}{16}) + \frac{49}{8} - \frac{8}{8}$

$= -2(x - \frac{7}{4})^2 + \frac{41}{8}$

vertex is: $(\frac{7}{4}, \frac{41}{8})$

General Form - Standard Form

$f(x) = ax^2 + bx + c$

$f(x) = a(x-h)^2 + k$

the value of "a" will be the same when you move from one form to another

you can now do:

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