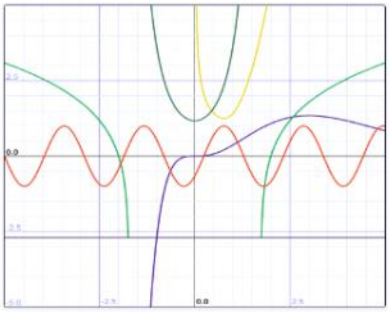
Lesson 4 Word Problems

Date:

Chapter 4: Linear and Quadratic Functions:



Lesson 4:

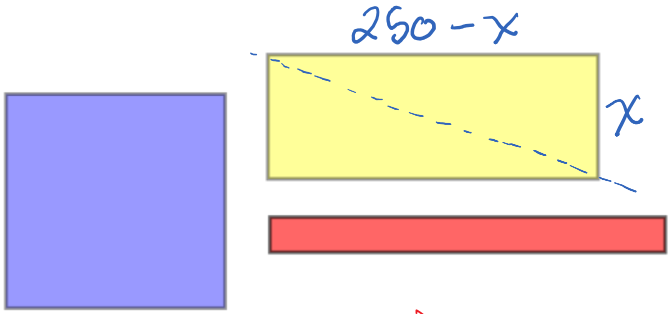
Word Problems

Maximum and **Minimum** problems examine situations where it is desirable to find the maximum or the minimum of a quadratic function

therefore it is desirable to find the **vertex** of these functions

Example 1

You have a 500-foot roll of fencing and a large field. You want to construct a rectangular playground area. What is the **largest** area that you can construct?



4 sides = 500m

2 sides = 250 m

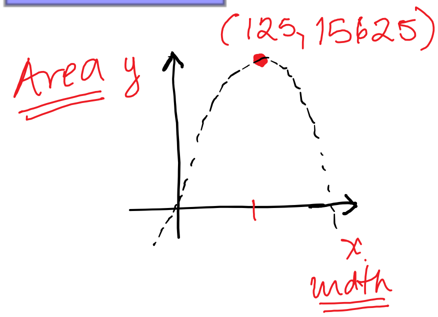
$\therefore A = l \times w$

$A = (250 - x)(x)$

$A = 250x - x^2$

$A = -x^2 + 250x$

parabola



$A = -1(x^2 - 250x) + 0$

$= -1(x - 250x + 15625 - 15625) + 0$

$= -1(x - 125)^2 + 15625$

$\boxed{\text{largest area: } 125 \times 125 \text{ ft}}$

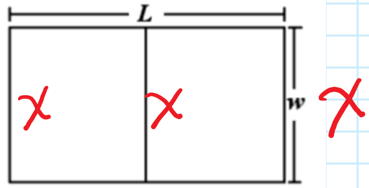
What conjecture could you make concerning rectangular enclosures and maximum area?

SQUARES have the largest area with a fixed amount of "fencing"

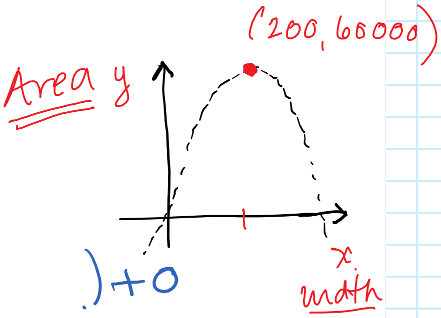
Example 2

You have a 1200-foot roll of fencing and a large field. You want to make two rectangular paddocks by splitting a rectangular enclosure in half. What are the dimensions of the largest such enclosure?

$$L = \frac{1200-3x}{2}$$



$$A = \left(\frac{1200-3x}{2}\right)(x)$$
$$= (600-1.5x)(x)$$
$$= 600x - 1.5x^2$$
$$= -1.5x^2 - 600x$$
$$A = -1.5(x^2+400x \qquad)+0$$
$$= -1.5(x^2+400x+40000-40000)+0$$
$$= -1.5(x+200)^2+60000$$

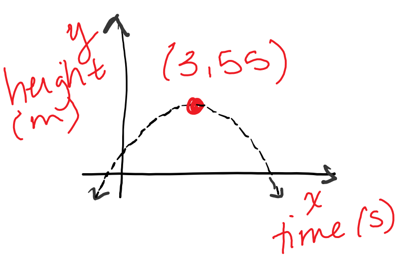


$\boxed{\text{largest area : 200 x 300 ft}}$

Example 3

A model rocket is launched from the roof of a building. Its flight path is described by $h(t) = -5t^2 + 30t + 10$ where h(t) is the height of the rocket above the ground in metres and t is the time after the launch in seconds.

What is the rocket's maximum height? When does it occur?



$$f(x) = -5t^2+30t+10$$
$$= -5(t^2-6t \qquad)+10$$
$$= -5(t-6t+9-9)+10$$
$$= -5(t-3)^2+10+45$$
$$= -5(t-3)^2+55$$

$\boxed{\therefore \text{ max height is 55 m at 3 seconds}}$

Example 4

Your factory produces lemon-scented widgets.

You know that the cost to produce each widget (unit cost) becomes less, the more you produce (for a time). But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock, adding a "night shift", increased insurance, etc.

The employee in accounting says that your unit cost for producing x thousands of units a day can be approximated by the formula:

$$C = 0.0023125\,x^2 - 0.185\,x + 12.6$$

Find the daily production level that will minimize your unit cost.

need to "see" the parabolic function

Cost ($) is a function of # widgets

[redacted]

completing the square
is messy!

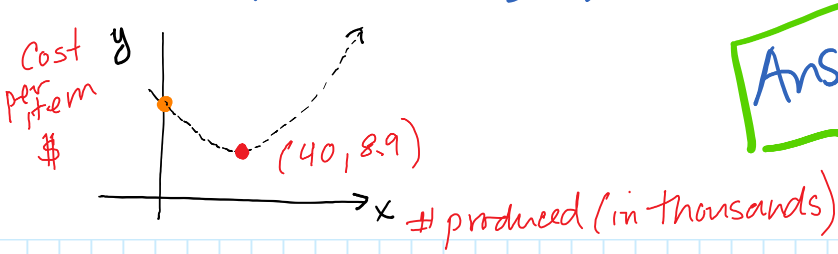
$$C = 0.0023125x^2 - 0.185x + 12.6$$

$$= .0023125(x^2 - 80x \qquad) + 12.6$$

$$= .0023125(x^2 - 80 + 1600 - 1600) + 12.6$$

$$= .0023125(x-40)^2 + 12.6 - 3.7$$

$$= .0023125(x-40)^2 + 8.9$$



Ans: 40000 widgets

Example 5

For a summer job, you run a canoe-rental business on a small lake in Northern Ontario. You currently charge \$12 per canoe and at that price you attract an average of 36 customers per day. You have done some market research and have learned that, for every \$1.50 increase in rental price, your business can expect to lose two rentals per day. **Use this information to determine how much you should be charging in order to maximize your income.**

try making a chart

- number of price hikes x
- cost per rental
- number of rentals
- revenue

2

$I = (12+1.5x)(36-2x)$

$= 432 - 24x + 54x - 3x^2$

$= -3x^2 + 30x + 432$

$= -3(x^2 - 10x + 25 - 25) + 432$

$= -3(x-5)^2 + 507$

$\boxed{\therefore \text{ 26 rentals}}$

1

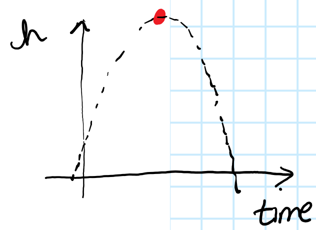
x	charge per canoe \$	# of rentals	I \$ income
0	12	36	432
1	13.50	34	459
2	15	32	480
x	$12+1.5x$	$36-x$	$(12+1.5x)(36-2x)$

Example 6

A water balloon is catapulted into the air so that its height h, in metres, after t seconds is $h(t) = -4.9t^2 + 27t + 2.4$

How high is the balloon after 1 second?

$h(1) = -4.9(1)^2 + 27 = \boxed{24.5\text{ m}}$



What is the max. height reached by the balloon? When does this occur?

$h(t) = -4.9(t^2 - 5.5102t \quad) + 2.4$

$= -4.9(t^2 - 5.5102t + 7.59059) + 2.4 + 37.1939$

$= -4.9(t - 2.7551)^2$

$\boxed{\text{after 2.76 seconds it is @ 39.594 m}}$