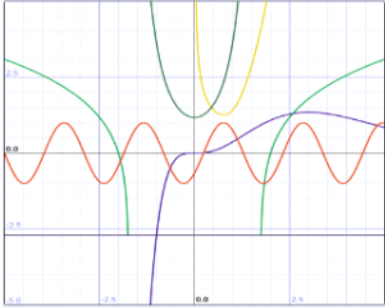


Lesson 9 Solving Quadratic Inequalities

Date:

Chapter 4: Linear and
Quadratic Functions:



Lesson 9:

Solving
Quadratic
Inequalities

what is the solution to the following equality?

$$2x+3=11$$

$$2x = 11 - 3$$

$$2x = 8$$

$$x = 4$$

$$2x + 3 = 11 \quad 2x + 3 - 11 = 0$$

$$2(4) + 3 = 11 \quad 2(4) + 3 - 11 = 0$$

$$8 + 3 = 11 \quad 8 + 3 - 11 = 0$$

$$11 = 11 \quad 0 = 0$$

$x = 4$

solution to both problems is exactly the same $x = 4$

rearranging and making it equal to zero did not change the solution to the original problem

what is the solution to the following inequalities?

$$2x + 3 > 11 \quad \textcircled{1} \quad 2x > 8 \quad \text{subtract 3 from each side}$$

$$\textcircled{2} \quad 2x - 8 > 0 \quad \text{or subtract 11 from both sides}$$

again, for each of these, the solution is exactly the same...

$$x > 4$$

ex

$$2(4.5) > 8$$

$$9 > 8 \quad \checkmark$$

$$\text{ex} \quad 2(4.5) - 8 > 0$$

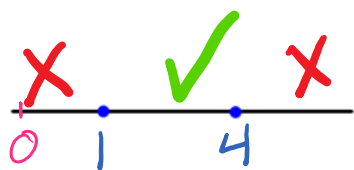
$$9 - 8 > 0$$

$$1 > 0 \quad \checkmark$$

Rearranging an inequality will not change the solution to the original

ex 1 Determine the solution set of the following inequality:

$$-2x^2 + 10x - 12 \geq -4$$



STEPS

1. pretend the inequality is an equal sign
2. make the equation equal to 0
3. Solve for x (ZPP, QF, etc)
4. place solutions for x on a number line
5. TWO SOLUTIONS: choose a number form one of the sections on the number line...test in the inequality



$$-2x^2 + 10x - 8 = 0 \quad \div \text{ b/s by } -2 \text{ to make it easier ;}$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$x-4=0$	$x-1=0$
$x=4$	$x=1$

test 0

$$-2(0)^2 + 10(0) - 12 \geq -4$$

$$0 + 0 - 12 \geq -4$$

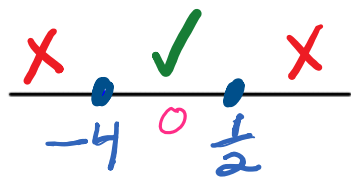
$$-12 \geq -4 \quad \text{FALSE X}$$

$$\therefore -2x^2 + 10x - 12 \geq -4 \quad \text{when } x \in [1, 4]$$

ex 2

Determine the solution set of the following inequality

$$(3x+12)(2x-1) \leq 0$$



STEPS

1. pretend the inequality is an equal sign
2. make the equation equal to 0
3. Solve for x (ZPP, QF, etc)
4. place solutions for x on a number line
5. TWO SOLUTIONS: choose a number form one of the sections on the number line...test in the inequality



$$(3x+12)(2x-1) = 0$$

$3x+12=0$	$2x-1=0$
$3x=-12$	$2x=1$
$x=-4$	$x=\frac{1}{2}$

test 0

$$(3x+12)(2x-1) \leq 0$$

$$(3(0)+12)(2(0)-1) \leq 0$$

$$(12)(-1) \leq 0$$

$$-12 \leq 0 \quad \text{TRUE } \checkmark$$

$$x \in [-4, \frac{1}{2}]$$

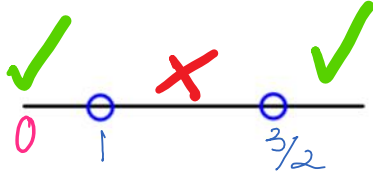
$$x \in [-4, \frac{1}{2}]$$

$$-12 = 0 \text{ TRUE}$$

ex 3

Determine the solution set of the following inequality

$$2x^2 - 5x > -3$$



$$2x^2 - 5x + 3 = 0$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(x-1)(2x-3) = 0$$

$$\begin{array}{l|l} x-1=0 & 2x-3=0 \\ x=1 & x=\frac{3}{2} \end{array}$$

STEPS

1. pretend the inequality is an equal sign
2. make the equation equal to 0
3. Solve for x (ZPP, QF, etc)
4. place solutions for x on a number line
5. TWO SOLUTIONS: choose a number from one of the sections on the number line...test in the inequality

$$\checkmark \text{ X } \checkmark \text{ or } \text{ X } \checkmark \text{ X}$$

test 0

$$2(0)^2 - 5(0) > -3$$

$$0 > -3 \text{ TRUE } \checkmark$$

$$x \in (-\infty, 1 \cup]\frac{3}{2}, \infty)$$

ex 4

Determine the solution set of the following inequality

$$2(x+3)^2 - 8 \leq 90$$

$$2(x+3)^2 = 90 + 8$$

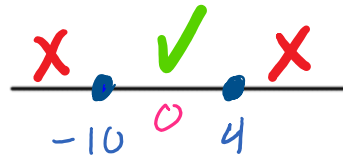
$$(x+3)^2 = 49$$

$$x+3 = \pm 7$$

$$x+3 = 7 \quad x+3 = -7$$

$$x_1 = 4$$

$$x_2 = -10$$



$$\checkmark \text{ X } \checkmark \text{ or } \text{ X } \checkmark \text{ X}$$

$$x \in [-10, 4]$$

test 0 $2(0+3)^2 - 8 \leq 90$

$$2(9) - 8 \leq 90$$

$$10 \leq 90 \text{ TRUE } \checkmark$$

ex 5

$$3x^2 - 10x - 2 > 2$$

$$3x^2 - 10x - 4 = 0$$

$$a = 3$$

$$b = -10$$

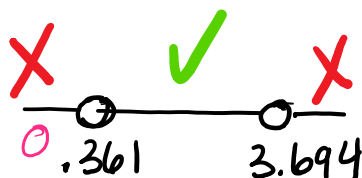
$$c = -4$$

$$\Delta = b^2 - 4ac$$

$$= (-10)^2 - 4(3)(-4)$$

$$= 100 + 48$$

$$= 148 \quad \Delta^+ \quad \therefore 2 \text{ solutions}$$



test 0

$$3(0)^2 - 10(0) - 2 > 2$$

$$-2 > 2 \quad \text{FALSE}$$

X

$$x = \frac{10 \pm \sqrt{148}}{6}$$

$$x_1 = 3.694 \quad x_2 = 0.361$$

$$x \in]0.361, 3.694[$$

ex 6

$$3(x-7)^2 + 10 \geq 19$$

$$3(x-7)^2 = 9$$

$$(x-7)^2 = 3$$

$$x-7 = \pm \sqrt{3}$$

$$x = +\sqrt{3} + 7 \quad x = -\sqrt{3} + 7$$

$$= 8.73$$

$$= 5.27$$

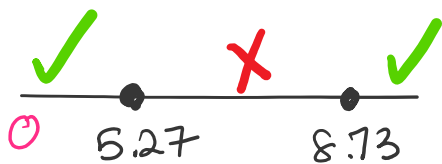
test 0

$$3(0-7)^2 + 10 \geq 19$$

$$3(49) + 10 \geq 19$$

$$147 + 10 \geq 19$$

$$\text{TRUE } 157 \geq 19 \quad \checkmark$$



$$x \in]-\infty, 5.27] \cup [8.73, \infty[$$

ex 7

$$3x^2 + 7x - 10 < 4$$

$$3x^2 + 7x - 10 - 4 = 0$$

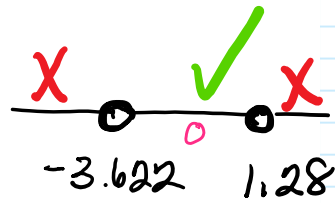
$$3x^2 + 7x - 14 = 0$$

$$\begin{aligned} a &= 3 & \Delta &= b^2 - 4ac \\ b &= 7 & &= 49 - 4(3)(-14) \\ c &= -14 & &= 217 \end{aligned}$$

$\Delta >$
 \therefore 2 solutions

$$x = \frac{-7 \pm \sqrt{217}}{6}$$

$$x = 1.28 \quad x = -3.622$$



test 0

$$\begin{aligned} 3(0)^2 + 7(0) - 10 &< 4 \\ 0 - 10 &< 4 \\ -10 &< 4 \quad \text{TRUE} \end{aligned}$$

$$x \in]-3.622, 1.28[$$

What are we missing so far???

cases where there is :

ONE solution for x
ZERO solutions for x

ex 8

$$(x+3)^2 \geq 0$$

$$x+3=0$$

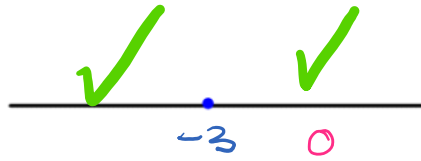
$$x=-3$$

test 0

$$(0+3)^2 \geq 0$$

$$3^2 \geq 0$$

$$\checkmark 9 \geq 0 \text{ TRUE}$$



$\checkmark \times \checkmark$ or $\times \checkmark \times$

doesn't really apply:
either both sections are $\checkmark \checkmark$
or both are $\times \times$

$$x \in \mathbb{R}$$

ex 9

$$3(x-4)^2 + 5 < 5$$

$$3(x-4)^2 + 5 = 5$$

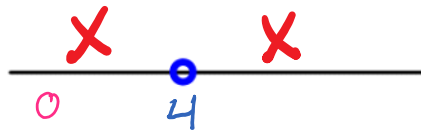
$$3(x-4)^2 = 0$$

$$(x-4)^2 = 0$$

$$x-4=0$$

$$x=4$$

$$x \in \emptyset$$



$\checkmark \times \checkmark$ or $\times \checkmark \times$

test 0

$$3(0-4)^2 + 5 < 5$$

$$3(16) + 5 < 5$$

$$48 + 5 < 5$$

$$53 < 5 \quad \times$$

FALSE

ex 10

$$-2(x+6)^2 - 10 \leq 4$$

$$-2(x+6)^2 = 14$$

$$(x+6)^2 = -7$$

No solution

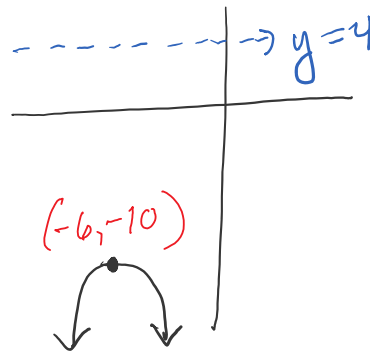
test 0

$$-2(0+6)^2 - 10 \leq 4$$

$$-2(36) - 10 \leq 4$$

$$-72 - 10 \leq 4$$

$$\checkmark -82 \leq 4$$



$$x \in \mathbb{R}$$

ex 11

$$2(x-3)^2 + 8 \leq -6$$

$$2(x-3)^2 = -6 - 8$$

$$2(x-3)^2 = -14$$

$$(x-3)^2 = -7$$

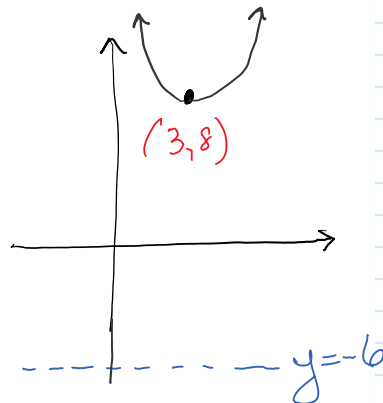
No solution

test 0

$$2(0-3)^2 + 8 \leq -6$$

$$2(-3)^2 + 8 \leq -6$$

$$\times 26 \leq -6 \text{ FALSE}$$



$$x \in \emptyset$$

TB1: p.217

3 Solve the following inequalities using the algebraic method.

- a) $(2x - 1)(x + 3) > 0$ b) $(1 - 2x)(4 - 2x) < 0$ c) $(4x + 10)(2x - 7) \leq 0$
d) $x^2 - 64 > 0$ e) $-x^2 + 2x - 1 < 0$ f) $5x^2 < 8x + 4$
g) $x^2 + 5x \geq -6$ h) $x(x + 6) + 9 \geq 16$ i) $(x - 1)(x + 2) > 4$

4 Solve each of the following inequalities using the graphical method.

- a) $x^2 + 5x + 6 > 0$ b) $2x^2 - 4x + 2 < 0$ c) $-x^2 - 1 \leq 0$
d) $-3x^2 + 2x < 1$ e) $2(-1.5x^2 + x + 5) \geq 10$ f) $2x^2 \geq 4x + 3$

-
- 3. a)** $]-\infty, -3[\cup]\frac{1}{2}, +\infty[$ **b)** $]\frac{1}{2}, 2[$ **c)** $[\frac{-5}{2}, \frac{7}{2}]$
d) $]-\infty, -8[\cup]8, +\infty[$ **e)** $]-\infty, 1[\cup]1, +\infty[$ **f)** $]-\frac{2}{5}, 2[$ **ANSWERS**
g) $]-\infty, -3[\cup]-2, +\infty[$ **h)** $]-\infty, -7[\cup]1, +\infty[$ **i)** $]-\infty, -3[\cup]2, +\infty[$
4. a) $]-\infty, -3[\cup]-2, +\infty[$ **b)** \emptyset **c)** \mathbb{R}
d) \mathbb{R} **e)** $[0, \frac{2}{3}]$ **f)** $]-\infty, \frac{2 - \sqrt{10}}{2}] \cup [\frac{2 + \sqrt{10}}{2}, +\infty[$

you can now do:

WB

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