Lesson 6 Two Optimal Points and Dotted Line

Chapter 2: Optimization

Lesson 6: Two Optimal Points and Dotted Lines

A chicken and turkey breeder raises fewer than 10 000 birds each year.

She produces at least 4 times as many chickens as turkeys. She will raise at least 6000 chickens and 1000 turkeys.

Her profits are $2.00 per chicken and $7.00 per turkey.

How many of each type of bird should she raise this year to maximize profits?

Variables: let x be.... let y be....

Constraints Objective function

more chickens than turkeys
∴ # of chickens is 4 times more than turkeys

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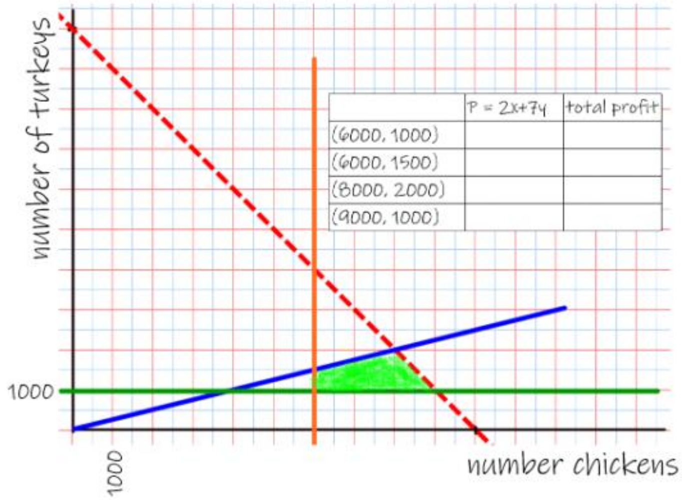
How many of each type of bird should she raise this year to maximize profits?

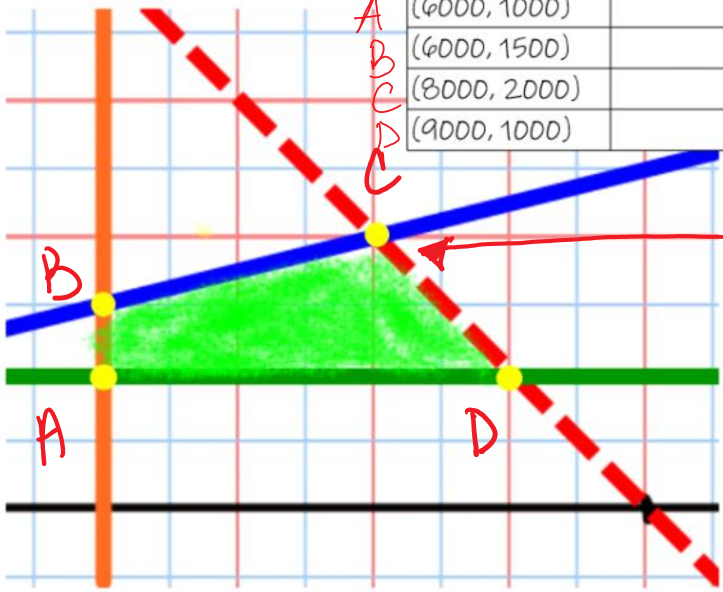
let x be number of chickens let y be number of turkeys

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y < 10000 \\ x \geq 6000 \\ y \geq 1000 \\ x \geq 4y \end{cases}$$

$$P = 2x + 7y$$


number of turkeys
1000
1000
number chickens
P = 2x+7y
total profit
(6000, 1000)
(6000, 1500)
(8000, 2000)
(9000, 1000)

	P = 2x+7y	total profit
A (6000, 1000)		19000
B (6000, 1500)		22500
C (8000, 2000)		30000
D (9000, 1000)		25000



Maximum Point is on a broken line.
∴ C is not a good solution

When the optimal solution is not viable (vertex falls on a **broken line**), then choose a point that is **inside** the polygon, closest to the vertex.

You **MUST verify** that this point meets the constraints...algebraically!

Try (8000, 1999) but test it in all the constraints

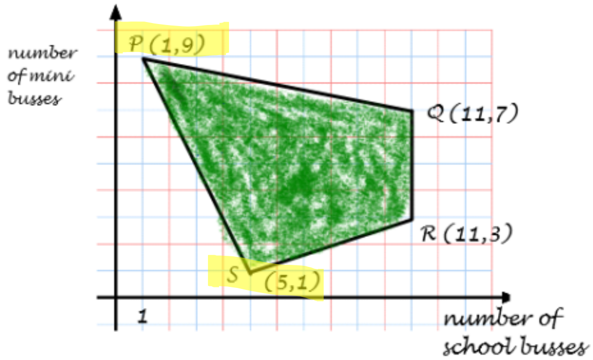
you must show the TEST for all points →

$$\begin{cases} x \geq 0 & 8000 \geq 0 \checkmark \\ y \geq 0 & 1999 \geq 0 \checkmark \\ x + y < 10000 & 8000 + 1999 < 10000 \checkmark \\ x \geq 6000 & 8000 \geq 6000 \checkmark \\ y \geq 1000 & 1999 \geq 1000 \checkmark \\ x \geq 4y & 8000 \geq 4(1999) \checkmark \end{cases}$$

Since (8000, 1999) meets all the constraints, it is the optimum solution $\therefore$ $P = 2(8000) + 7(1999)$
$= \$29\,993$ for 8000 chickens + 1999 turkeys

A school wants to minimize the transportation costs involved in taking students to Mont Megantic. The following polygon of constraints represents the solutions for this situation. Each school bus costs $80 and each minibus costs $40. The school needs to determine the minimum transportation cost.

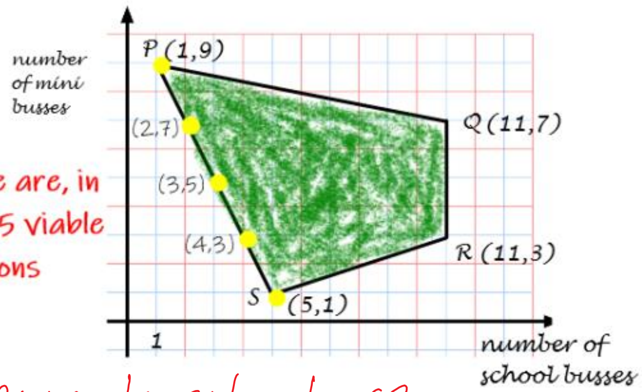
In this situation, how many solutions minimize the transportation cost involved in taking students on this field trip?



Vertices of the polygon of constraints	$C = 80x + 40y$
P (1, 9)	$440
Q (11, 7)	$1 160
R (11,3)	$1 000
S (5, 1)	$440

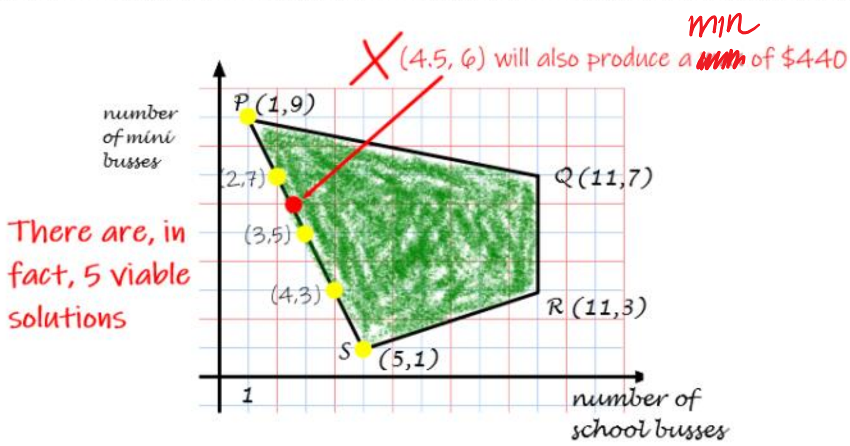
MINIMIZE!

If the optimal solution (maximum or minimum) is achieved at two consecutive vertices (P and S), then **each VIABLE point along the edge PS** is also an optimal solution.



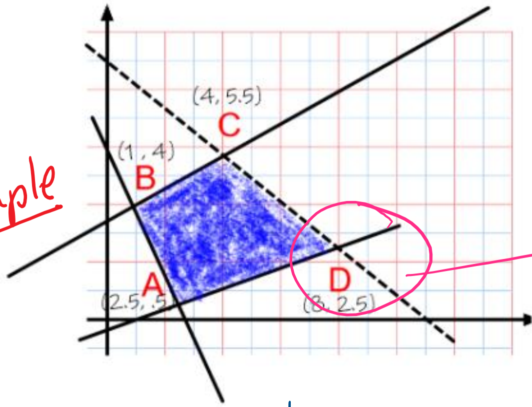
There are, in fact, 5 viable solutions

Be sure to only choose integers if the situation demands integer solutions (ie people, puppies, busses)


(4.5, 6) will also produce a min of $440
There are, in fact, 5 viable solutions
number of mini busses
P (1,9)
(2,7)
(3,5)
(4,3)
S (5,1)
Q (11,7)
R (11,3)
1
number of school busses

When your MAX is not on a solid line.....

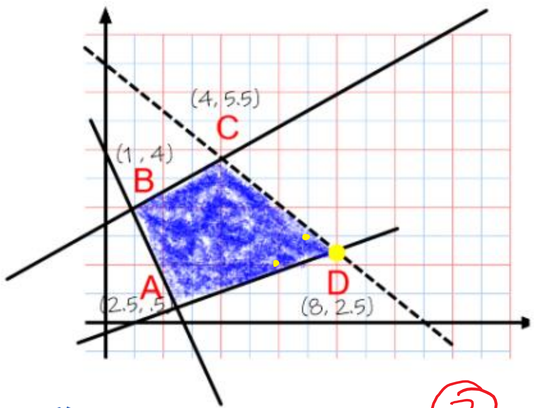
example



	Otpimization Equation P=12.5x + 15.3y
A	38.90
B	73.70
C	134.15
D	~~138.25~~

you are seeking a maximum
you are seeking integer solutions only.

When your MAX is not on a solid line.....



	Otpimization Equation P=12.5x + 15.3y
A	38.90
B	73.70
C	134.15
D	138.25
(6,2) E	105.6
(7,3) F	133.4

① FIND RULES

② New points

line CD

slope $\frac{2.5-5.5}{8-4} = \frac{-3}{4}$

$y = -\frac{3}{4}x + b$

$5.5 = -.75(4) + b$

$5.5 + 3 = b$

$8.5 = b$

$y < -.75x + 8.5$

line AD

slope $= \frac{2.5-.5}{8-2.5} = \frac{2}{5.5}$

$y = \frac{2}{5.5}x + b$

$2.5 = \frac{2}{5.5}(8) + b$

$-\frac{9}{22} = b$

$y \geq \frac{2}{5.5}x - \frac{9}{22}$

③ test new point (7,3)

→ $3 < -.75(7) + 8.5$

$3 < 3.25$ ✓

→ $3 \geq \frac{2}{5.5}(7) - \frac{9}{22}$

$3 \geq \frac{47}{22}$ ✓

∴ (7,3) is the optimal solution

you can now do:

WB

- Page 55 Activity 3 a) b) f) #6
- Page 58 and 59 #6 - 8