Name : ___________________________________

Group : ___________________________________

Date : ___________________________________

568536 - Mathematics

Question Booklet

1 Jonathan works at a golf club during his summer vacation. He sometimes cleans the premises and sometimes works in the kitchen at the club's restaurant.

Jonathan makes \$8 per hour when cleaning the premises and \$9.50 per hour when working in the kitchen.

There are certain constraints on the number of hours he can devote to each job every week. This situation is represented by the system of inequalities and the polygon of constraints given below.

$$x \geq 0$$

$$y \geq 0$$

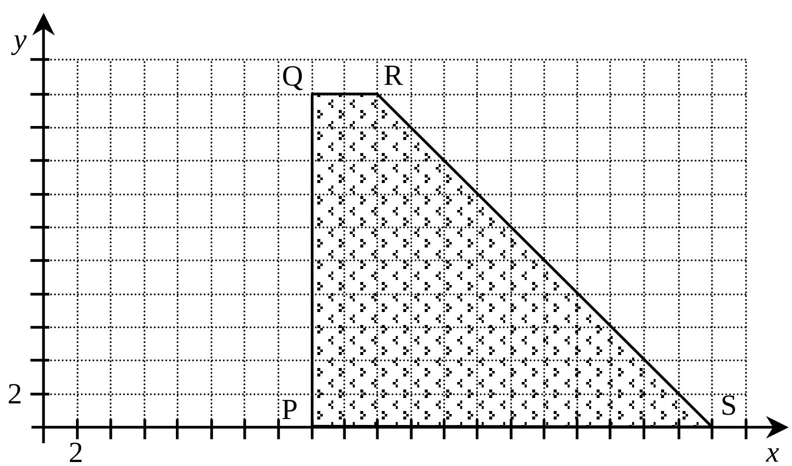
$$x + y \leq 40$$

$$x \geq 16$$

$$y \leq 20$$

x : the number of hours spent cleaning the premises

y : the number of hours spent working in the kitchen



Coordinates of the vertices of the polygon
P(16, 0)
Q(16, 20)
R(20, 20)
S(40, 0)

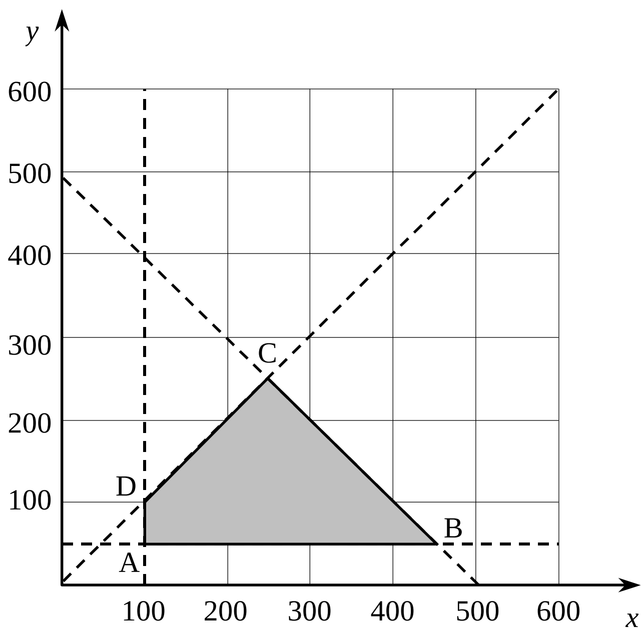
This week, Jonathan's employer informed him that there would be an additional constraint. This new constraint is represented by the following inequality: $x \geq y + 20$

With this new constraint, by how much will Jonathan's maximum possible income decrease?

2

To raise money, the Graduation Committee decides to sell cases of fruit. The following polygon represents the constraints that must be respected.

If *x* represents the number of cases of oranges for sale and *y*, the number of cases of grapefruit for sale, the constraints are:



① $x \leq y$

② $x + y \leq 500$

③ $x \geq 100$

④ $y \geq 50$

For each case of oranges and grapefruit sold, the Graduation Committee makes a profit of $1.00 and $1.50, respectively.

Yesterday, the head of the committee received a call from the supplier. Because of a recent flood, the supplier can deliver a maximum of 400 cases of fruit.

By how much will the maximum possible revenue decrease because of the flood?

3 Six inequalities are represented by the polygon of constraints below.

$l_1 \quad x \geq 0$

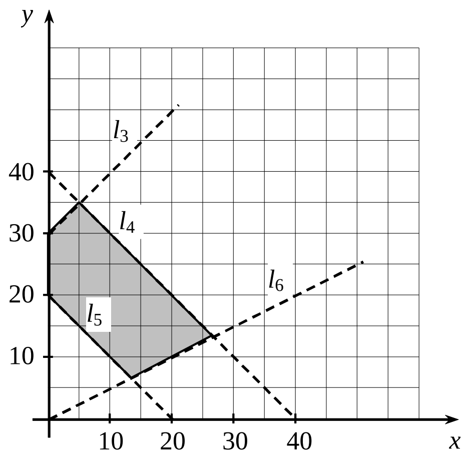
$l_2 \quad y \geq 0$

$l_3 \quad ?$

$l_4 \quad x + y \leq 40$

$l_5 \quad x + y \geq 20$

$l_6 \quad y \geq \dfrac{x}{2}$



l_3 forms a right angle with l_4.

Which of the following is the missing in equality (l_3)?

A) $x + y \leq 30$

B) $x + y \geq 30$

C) $y - x \geq 30$

D) $y - x \leq 30$

4 The city council of a town wants to minimize the cost of staffing its recreation centres in the summer months. The council has determined that supervisors will be paid $3500 for the summer and staff workers will be paid $1500 for the summer.

The council wants to hire its employees using the following constraints:

- The maximum number of employees for its centres is 30 and the minimum is 18.
- The council also wants to hire at least 6 supervisors but no more than 14 supervisors.
- It wants to hire at least 8 staff workers.
- The number of staff workers must be at most twice the number of supervisors.

How many staff workers and how many supervisors can the town council hire and minimize its costs?

5 Wheeler is a producer of mountain bikes and road bikes. Because of its small size, it can build no more than 80 bikes each week. To meet certain conditions in its workshop, it must build at least 45 mountain bikes, and at least 10 road bikes weekly. To meet consumer demand, it must manufacture at least 3 times as many mountain bikes as road bikes.

The following is the system of constraints for Wheeler's weekly bike production:

x = the number of road bikes produced weekly

y = the number of mountain bikes produced weekly

$x \geq 0$

$y \geq 0$

$x \geq 10$

$y \geq 45$

$x + y \leq 80$

$y \geq 3x$

For each road bike and mountain bike produced, Wheeler earns a profit of $250 and $175, respectively.

What is the maximum weekly profit that can be earned?

6 A fisherman has to separate his daily catch of shellfish into two categories before he can sell them. Lobsters are sold for $8.70 each and crabs are sold for $9.60 each.

On an average day, the fisherman can expect to catch a minimum of 35 crabs and a maximum of 60. By experience, there are at most twice as many lobsters as crabs in a daily catch and never has the fisherman caught more than 140 shellfish in a single day.

Using a polygon of constraints, determine the maximum revenue that this fisherman can expect to mak

7 Instruments Quebecois makes two types of graphing calculators, the Gold Edition and the Bronze Edition. In order to meet daily demands, it must make at least 200 Gold Editions and at least 100 Bronze Editions.

The factory produces at least twice as many Gold Editions as Bronze Editions, but can make no more than 600 calculators a day.

Given x: number of Gold Editions

y: number of Bronze Editions

Constraints:

$x \geq 0$

$y \geq 0$

$x \geq 200$

$y \geq 100$

$x \geq 2y$

$x + y \leq 600$

The profit on a Gold Edition is \$20.00 and \$15.00 on a Bronze Edition.

In the answer booklet, graph the polygon of constraints and determine the number of calculators that the company should make to maximize its profit.

8 The Grad Committee plans to sell chocolate bars to raise money for its upcoming dance. This year the committee members have decided to sell two types, one with roasted almonds and the other with caramel. They have a maximum of 500 bars to sell. They expect to sell a minimum of 120 almond chocolate bars. From past experience, almond chocolate bars sell at most 4 times as well as caramel ones. They make a profit of \$0.80 for each almond chocolate bar and \$1 for each caramel chocolate bar.

Let x: number of almond chocolate bars y: number of caramel chocolate bars

What is the difference in the maximum profit if they had expected to sell a minimum of 160 almond chocolate bars rather than 120?

9 Murray plans a trip to New York in July. In order to save money, he works at two different part-time jobs on weekends. At the first job, he works a minimum of 10 hours per month and at the second, a maximum of 40 hours per month. Murray must work at least 30 hours per month but no more than 60 hours per month. He must work at least as many hours at the second job as he does at the first. He makes \$6.30 an hour at the first job and \$8 an hour at the second job.

Let *x*: number of hours per month at first job

y: number of hours per month at second job

The initial constraints for this situation are:

$$\begin{aligned} x &\geq 10 \\ y &\leq 40 \\ y &\geq 0 \\ x + y &\geq 30 \\ x + y &\leq 60 \\ y &\geq x \end{aligned}$$

Because of a shortage of employees, Murray was later advised that he could increase the number of hours he worked at the second job.

By how much did Murray's maximum possible salary increase because of the employee shortage?.

10 Kim is organizing a fundraiser for her soccer team. She will sell hot dogs and hamburgers outside a popular grocery store. She needs to purchase enough supplies to be able to make the following:

– at most 800 hot dogs and hamburgers

– at least 150 hot dogs

– a minimum of 100 but not more than 400 hamburgers

– at most twice as many hamburgers as hot dogs

Her cost is \$0.45 per hot dog and \$0.75 per hamburger. She will sell the hot dogs at \$1 each and hamburgers at \$1.50 each.

Let *x*: the number of hot dogs

y: the number of hamburgers

Given her constraints, how many hot dogs and hamburgers does Kim need to sell to make the greatest profit possible?

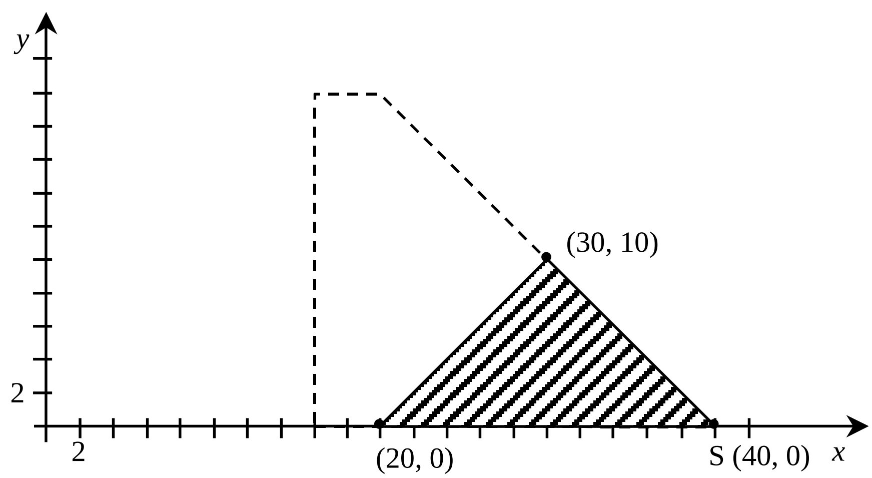
Optimization

1 Example of an appropriate method

Maximum possible income before the new constraint

Vertex	Income: $8x + 9.50\,y$
P (16, 0)	8 (16) + 9.50 (0) = \$128
Q (16, 20)	8 (16) + 9.50 (20) = \$318
R (20, 20)	8 (20) + 9.50 (20) = \$350 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = \$320

Vertices of the new polygon of constraints



Maximum possible income with the new constraint

Vertex	Income: $8x + 9.50\,y$
(20, 0)	8 (20) + 9.50 (0) = \$160
(30, 10)	8 (30) + 9.50 (10) = \$335 ← maximum income
S (40, 0)	8 (40) + 9.50 (0) = \$320

Difference between the two maximum possible incomes

$\$350 - \$335 = \$15$

Answer: With this new constraint, Jonathan's maximum possible income will decrease by $**15**.

Note: Students who used an appropriate method in order to determine the maximum possible income before **or** with the new constraint have shown that they have a partial understanding of the problem.

2

Example of an appropriate solution

Calculation of revenue before the new constraint is considered

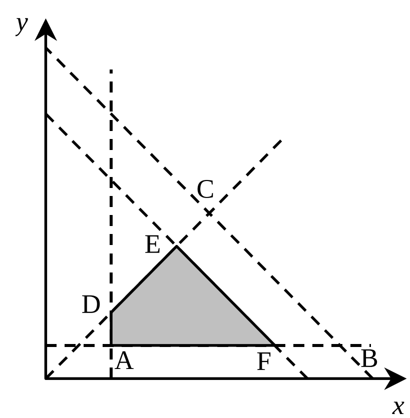
Vertices	R(*x*, *y*) = 1.00*x* + 1.50*y*
A(100, 50)	100 + 75 = $175
B(450, 50)	450 + 75 = $525
C(250, 250)	250 + 375 = $625
D(100, 100)	100 + 150 = $250

Maximum Revenue $\Rightarrow$ \$625   250 cases of oranges and

250 cases of grapefruit

Calculation of revenue after consideration of the constraint $x + y \leq 400$

Vertices	R(*x*, *y*) = 1.00*x* + 1.50*y*
A(100, 50)	$175
D(100, 100)	$250
E(200, 200)	$500
F(350, 50)	$425



Maximum Revenue $\Rightarrow$ 500 \$ 200 cases of oranges

200 cases of grapefruit

Decrease of revenue because of the flood

$625 - 500 = 125$

Answer: The decrease in revenue caused by the flood is $**125**.

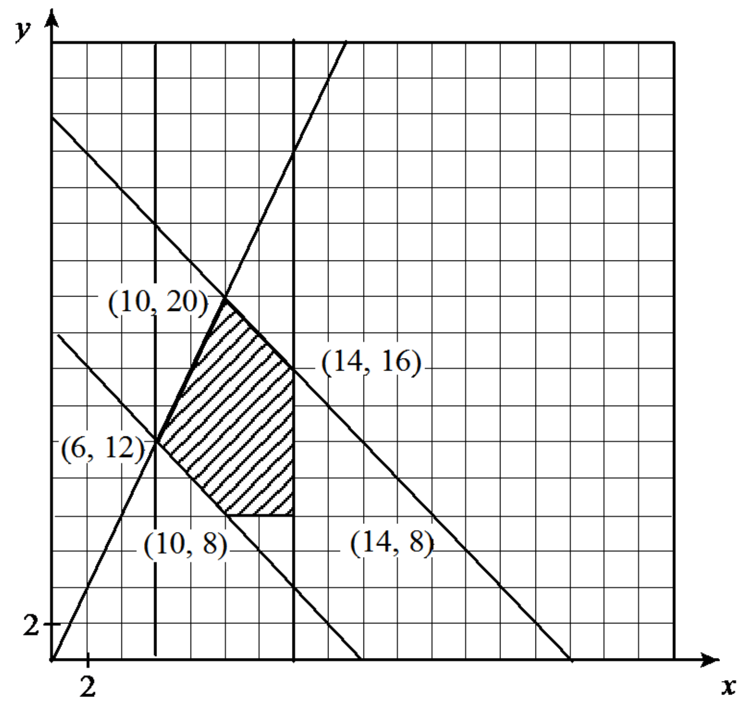
3 D

4

Example of an appropriate solution

x: number of supervisors

y: number of staff workers



Constraints : $x \geq 0$, $y \geq 0$

$x + y \leq 30$

$x + y \geq 18$

$x \geq 6$

$x \leq 14$

$y \geq 8$

$y \leq 2x$

Vertices of polygon of constraints:	(10, 8)	$\Rightarrow$	10(3500) + 8(1500) =	\$47 000
	(6, 12)	$\Rightarrow$	6(3500) + 12(1500) =	\$39 000
	(10, 20)	$\Rightarrow$	10(3500) + 20(1500) =	\$65 000
	(14, 16)	$\Rightarrow$	14(3500) + 16(1500) =	\$73 000
	(14, 8)	$\Rightarrow$	14(3500) + 8(1500) =	\$61 000

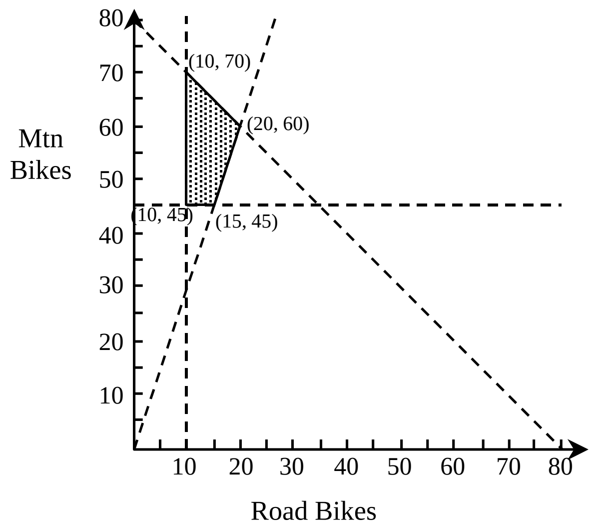
The minimum cost is \$39 000.

Answer The town should hire 6 supervisors and 12 staff workers in order to minimize its costs.

5 Example of an appropriate method

x = number of road bikes

y = number of mountain bikes



$x \geq 0$

$y \geq 0$

$x \geq 10$

$y \geq 45$

$x + y \leq 80$

$y \geq 3x$

Objective Function

Max. Profit = $250x + 175y$

Points (x, y)		Calculation	Profit
1.	(10, 45)	250(10) + 175(45)	\$10 375
2.	(15, 45)	250(15) + 175(45)	\$11 625
3.	(20, 60)	250(20) + 175(60)	\$15 500
4.	(10, 70)	250(10) + 175(70)	\$14 750

Answer The maximum weekly profit is $15 500.

6

Example of an appropriate method

Let x: number of lobsters

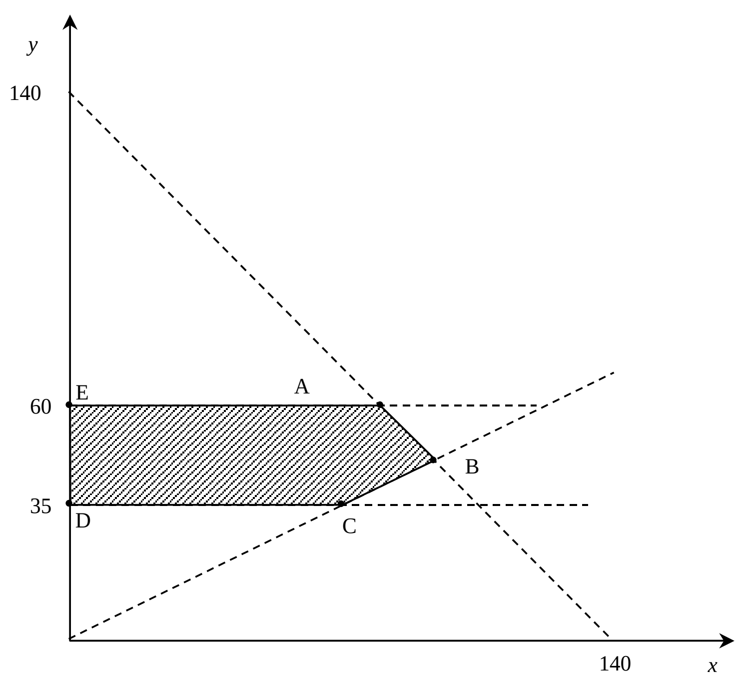
y: number of crabs

Constraints:

$$x \geq 0 \quad y \geq 0$$
$$y \geq 35$$
$$y \leq 60$$
$$x \leq 2y$$
$$x + y \leq 140$$

Objective Function: $R = 8.70x + 9.60y$

Graph:



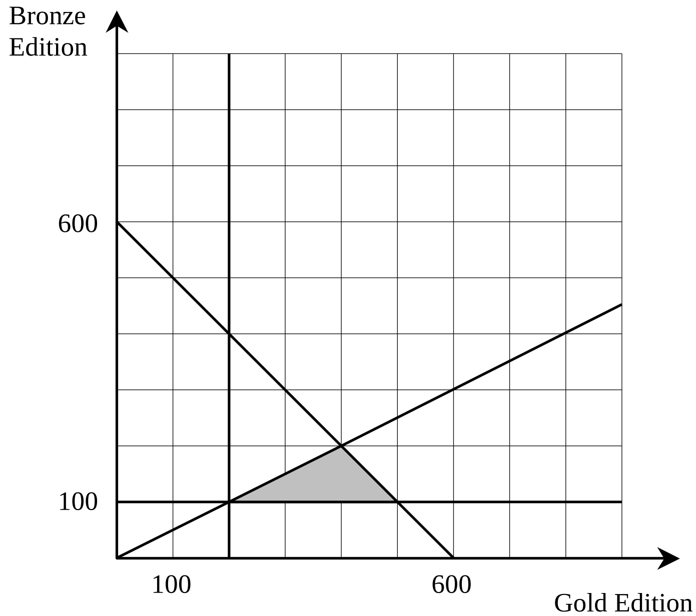
Vertex	$R = 8.70x + 9.60y$
A(80, 60)	1272 ← max
B($93.\overline{3}$, $46.\overline{6}$)	1259
C(70, 35)	945
D(0, 35)	336
E(0, 60)	576

Answer: The maximum revenue this fisherman can expect to make is **$1272**.

Note: Do not penalize students who did not include the non-negative constraints.

Students who determined the constraints and graphed the polygon have shown a partial understanding of the problem.

7 Example of an appropriate solution



Maximum possible profit

Vertex	Profit = $20x + 15y$
A(200, 100)	20(200) + 15(100) = $5500
B(400, 200)	20(400) + 15(200) = $11 000
C(500, 100)	20(500) + 15(100) = $11 500

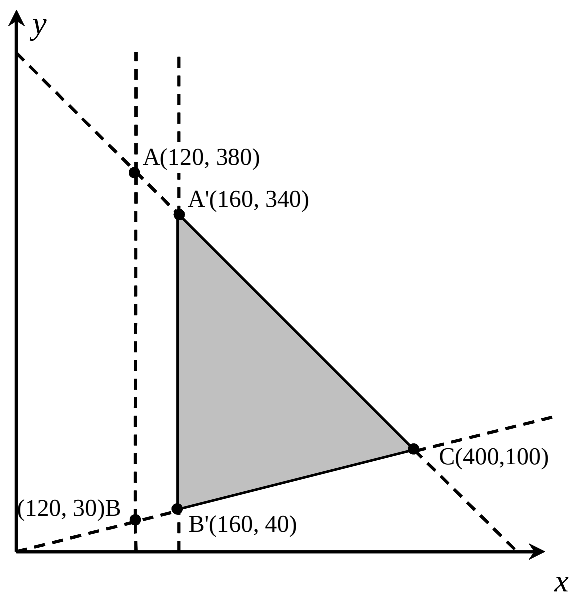
Answer: Instruments Quebecois should produce **500** Gold Editions and **100** Bronze Editions to maximize profit.

8

Example of an appropriate solution

x: number of almond chocolate bars

y: number of caramel chocolate bars



Constraints

$$x + y \leq 500$$
$$x \leq 4y$$
$$x \geq 120$$

Profit

P = 0.8x + y

Point	Profit
A(120, 380)	\$476
B(120, 30)	\$126
C(400, 100)	\$420

Point	Profit
A'(160, 340)	\$468
B'(160, 40)	\$168
C'(400, 100)	\$420

Difference in profit

\$476 – \$468 = \$8

Answer: The difference in the maximum profit is \$**8**.

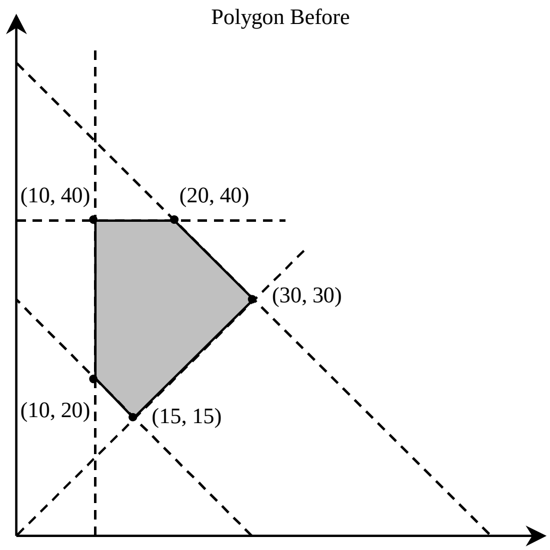
Note: Students who use an appropriate method in order to determine the constraints, graph the polygon and find the original corner points have shown they have a partial understanding of the problem.

9

Example of an appropriate solution

x: number of hours at first job per month

y: number of hours at second job per month

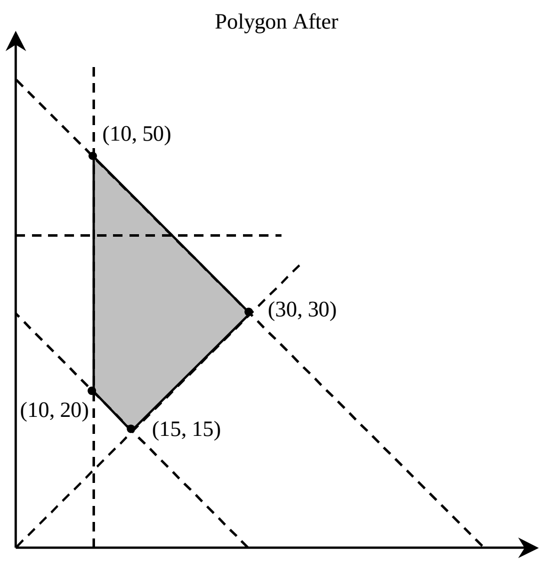


Constraints before

$$x \geq 10$$
$$y \leq 40$$
$$y \geq 0$$
$$x + y \geq 30$$
$$x + y \leq 60$$
$$y \geq x$$

Constraints after

$$x \geq 10$$
$$y \geq 0$$
$$x + y \geq 30$$
$$x + y \leq 60$$
$$y \geq x$$



Maximum Before

Vertices	S = 6.3x + 8y ($)
A(10, 40)	383
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
E(20, 40)	446

Maximum After

Vertices	S = 6.3x + 8y ($)
B(10, 20)	223
C(15, 15)	214.50
D(30, 30)	429
F(10, 50)	463

Difference in maximum salary

$463 – $446 = $17

Answer: Murray's maximum possible salary increased by $17

Note: Students who use an appropriate method in order to determine **the constraints, graph the original polygon, and find its vertices** have shown they have a partial understanding of the problem.

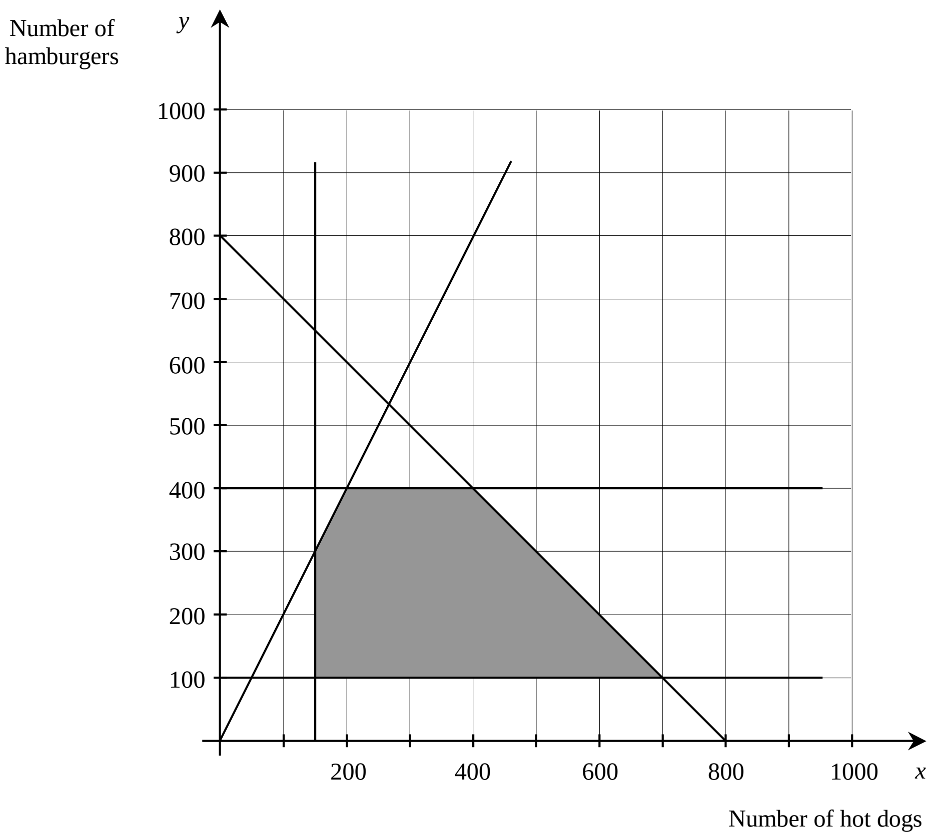
10 Example of an appropriate solution

x: number of hot dogs

y: number of hamburgers

Constraints

$$\begin{aligned} x + y &\leq 800 \\ x &\geq 150 \\ y &\geq 100 \\ y &\leq 400 \\ y &\leq 2x \end{aligned}$$



Points	P = $0.55x + 0.75y$
(150, 100)	157.5

(150, 300)	307.5
(200, 400)	410
(400, 400)	520
(700, 100)	460

Answer: Kim needs to sell **400** hot dogs and **400** hamburgers to make the greatest profit.

Note: Students who have found **the constraints** have shown they have a partial understanding of the problem.