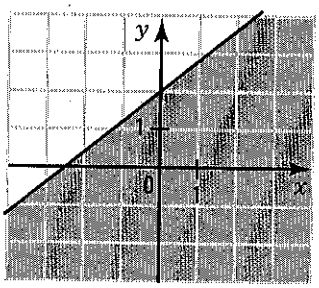
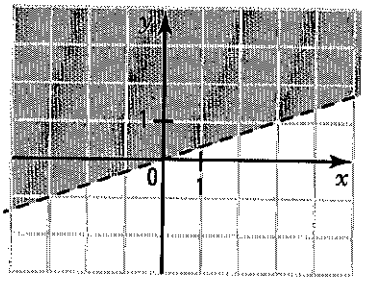
1. Represent graphically the solution set of the following inequalities.

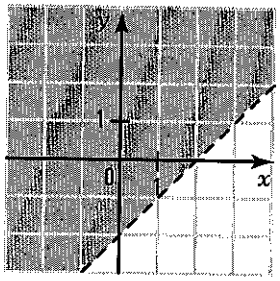
a) $-4x + 5y - 10 \leqslant 0$



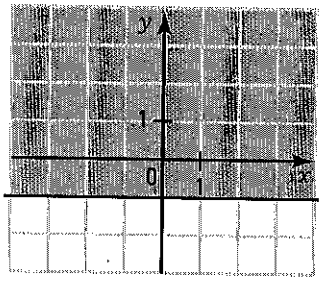
b) $x - 3y < 0$



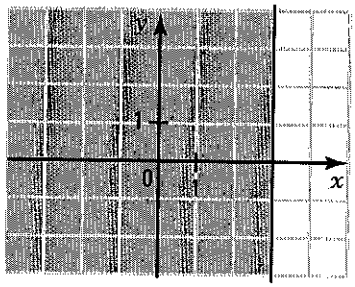
c) $y > x - 2$



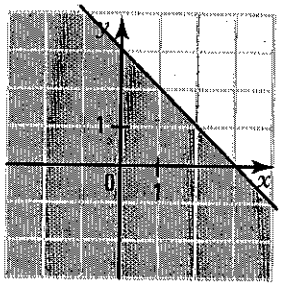
d) $y \geqslant -1$



e) $x \leqslant 3$



f) $x + y \leqslant 3$



2. Determine if the coordinates of the point P(3, –2) verify each of the following inequalities.

a) $5x - 4y > 10$ — *Yes*

b) $x \leqslant 4y$ — *No*

c) $x < 2y + 4$ — *No*

d) $-3x + 2y + 5 \leqslant 0$ — *Yes*

e) $x \leqslant 8$ — *Yes*

f) $\frac{x}{3} + \frac{y}{2} < 1$ — *Yes*

3. For each of the following situations,

1. identify the variables.
2. translate the situation into a two-variable first degree inequality.

a) The total number of boys and girls on a field trip is less than or equal to 150.

x: number of boys, y: number of girls; $x + y \leqslant 150$.

b) The perimeter of a rectangle is greater than 250 cm. ***x: length, y: width; $2x + 2y > 250$.***

c) At a summer camp, counsellors are paid $9.50 an hour and sports instructors are paid $15 an hour. The budget for these employees' salary is less than $9000.

x: number of counsellors, y: number of sports instructors; $9.50x + 15y < 9000$.

d) At a food products company, salad dressing is packaged in 100 ml bottles and 250 ml bottles. The total amount of dressing packaged in bottles is at least equal to 50 litres.

x: number of 100 ml bottles, y: number of 250 ml bottles; $100x + 250y \geqslant 50\,000$.

e) In a group of tourists, there are at most three times as many Francophones as there are Anglophones.

x: number of Francophones, y: number of Anglophones; $x \leqslant 3y$.

4. To raise money for their graduation party, secondary 5 students sell shirts and caps. Each shirt sells for \$15 and each cap sells for \$8. Translate each of the following constraints into a two-variable first degree inequality, knowing that x represents the number of shirts sold and y represents the number of caps sold.

a) The students want to raise at least \$850. $15x + 8y \geq 850$

b) They want to sell at most three times as many shirts as caps. $x \leq 3y$

c) They sold more than 70 items. $x + y > 70$

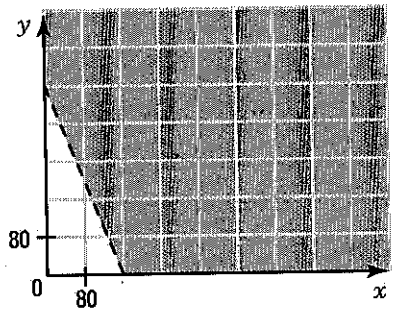
d) They sold a maximum of 40 shirts. $x \leq 40$

e) They sold at least as many shirts as caps. $x \geq y$

5. At a fundraising concert to help homeless people, organizers sell adult tickets for \$25 and student tickets for \$10. If x represents the number of adult tickets sold and y represents the number of student tickets sold, use a two-variable first degree inequality to translate each of the following statements and represent the solution set of the inequality in the Cartesian plane with an appropriate choice of scale.

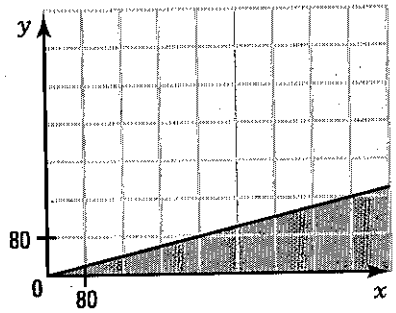
a) The organizers raised more than \$4000.

$25x + 10y > 4000$



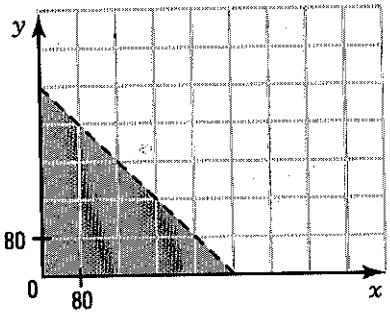
b) There were at least four times as many adult tickets sold as student tickets.

$x \geq 4y$



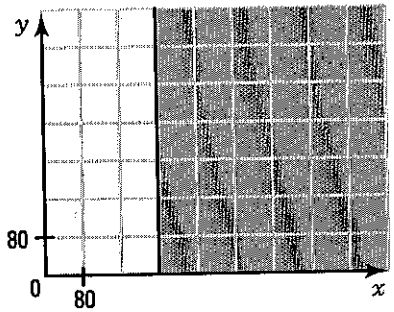
c) The number of tickets sold is less than 400.

$x + y < 400$



d) The number of adult tickets sold is greater than or equal to 240.

$x \geq 240$



6. For each of the following situations,

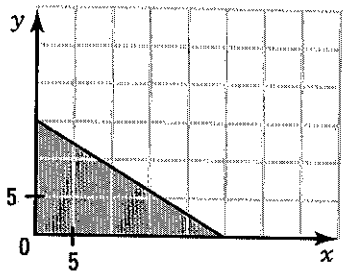
1. define the variables involved in the situation;
2. translate the situation into an inequality;
3. represent the situation in the Cartesian plane.

a) A garden has an area of 75 m². Each fruit patch occupies 3 m² and each vegetable patch occupies 5 m².

x: number of fruit patches,

y: number of vegetable patches;

$3x + 5y \leqslant 75$.

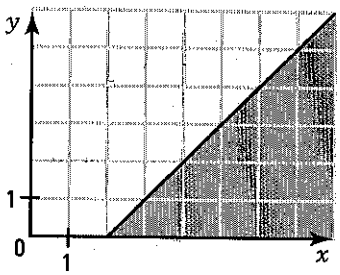


b) In Quebec's logging industry, timber production exceeds pulp and paper production by at least 2%.

x: percentage of timber production,

y: percentage of pulp and paper production;

$x \geqslant y + 2$.

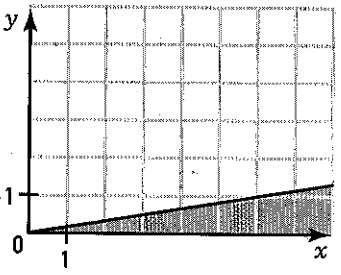


c) Quebec's tourist industry announces that there are at least 6 times as many tourists from Quebec as there are tourists from other parts of Canada.

x: number of tourists from Quebec,

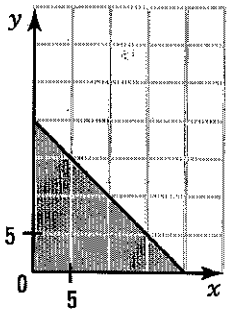
y: number of tourists from other parts of Canada;

$x \geqslant 6y$.



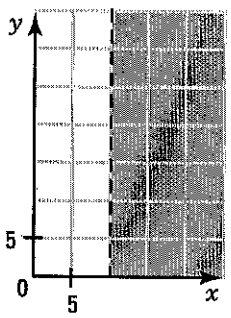
7. The manager of employees for a pharmaceutical company wishes to hire employees for the research department and employees for management. Research employees are paid \$40 an hour and management employees are paid \$16 an hour. If x represents the number of research employees and y the number of management employees, translate each of the following graphs into an inequality.

a)



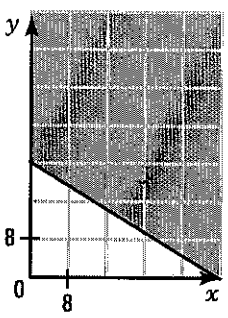
$x + y \leqslant 20$

b)



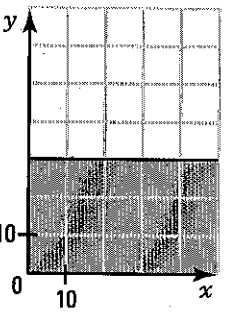
$x > 10$

c)



$3x + 5y \geqslant 120$

d)



$y \leqslant 30$

1. Solve the following systems using the appropriate method.

a) $\begin{cases} 3x + 2y = -5 \\ 5x + 3y = -7 \end{cases}$

S = {(1, -4)}

b) $\begin{cases} x = 3y - 8 \\ x = \frac{1}{2}y - 3 \end{cases}$

S = {(-2, 2)}

c) $\begin{cases} 3x + y = -4 \\ x = 2y - 13 \end{cases}$

S = {(-3, 5)}

d) $\begin{cases} y = -2x - 3 \\ 5x + y = -3 \end{cases}$

S = {(0, -3)}

e) $\begin{cases} y = 4x + \frac{1}{2} \\ y = 2x + 1 \end{cases}$

S = $\left\{\left(\frac{1}{4}, \frac{3}{2}\right)\right\}$

f) $\begin{cases} 4x + 3y = -28 \\ 3x - 2y = 13 \end{cases}$

S = {(-1, -8)}

2. In each of the following situations,

1. identify the variables;
2. write a system of two-variable first degree equations;
3. determine the solution of the system.

a) In a real estate project, there are three times as many condominiums as single-family houses. There is a total of 240 homes. How many condominiums are there?

x: number of condominiums

y: number of single-family houses

$\begin{cases} x = 3y \\ x + y = 240 \end{cases}$

There are 180 condominiums.

b) In a warehouse, there are 1250 boxes. Each small box occupies a volume of 7 dm^3 and each large box occupies a volume of 45 dm^3. The total volume occupied by the boxes is 42 950 dm^3. How many boxes of each size are there?

x: number of small boxes

y: number of large boxes

$\begin{cases} x + y = 1250 \\ 7x + 45y = 42\,950 \end{cases}$

There are 350 small boxes and 900 large boxes.

c) Determine the area of a rectangle if its length is 5 m more than twice its width and the perimeter of the rectangle is equal to 37 m.

x: length

y: width

$\begin{cases} x = 2y + 5 \\ 2x + 2y = 37 \end{cases}$

The area of the rectangle is equal to 63 m^2.

d) A car rental agency offers two options. The 1st one consists in paying a \$30 fixed amount and a \$0.08 amount per kilometre. The 2nd consists in paying a \$20 fixed amount and a \$0.10 amount per kilometre. Determine the number of kilometres that we must travel so that both options carry the same cost.

x: number of kilometres

y: rental cost

$\begin{cases} y = 0.08x + 30 \\ y = 0.10x + 20 \end{cases}$

The number of kilometres traveled so that the cost is the same is 500.

SYSTEM OF TWO-VARIABLE FIRST DEGREE INEQUALITIES

- A **system of two-variable first degree inequalities** is a system that can be written in the form:

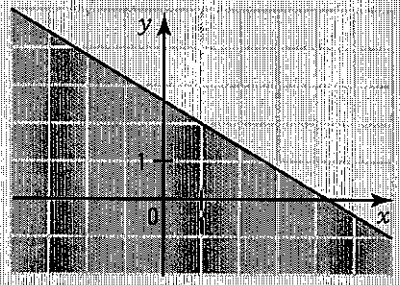
$$\begin{cases} a_1x + b_1y \geqslant c_1 \\ a_2x + b_2y \geqslant c_2 \end{cases} \qquad (\leqslant, >, <)$$

- The **solution set** of a of two-variable first degree inequalities is obtained by determining the intersection of the solution sets of each of the inequalities of the system.

Ex.: To solve the system $\begin{cases} 3x + 5y \leqslant 13 \\ x > 2y - 3 \end{cases}$, we proceed in the following way:

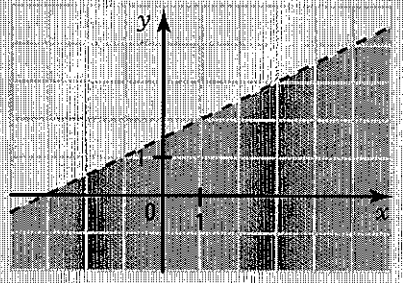
1 We represent the solution set of the inequality:

$3x + 5y \leqslant 13$

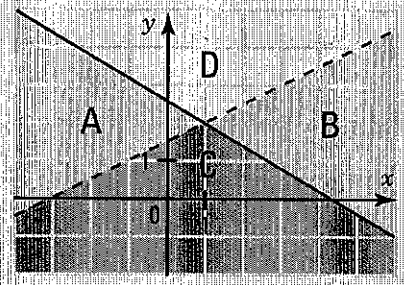


2 We represent the solution set of the inequality:

$x > 2y - 3$



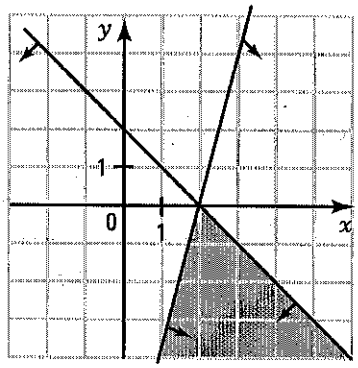
3 We deduce the solution set of the system.



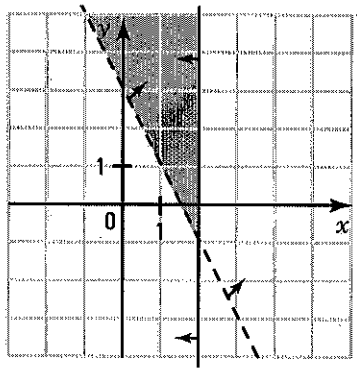
Any point belonging to region A verifies inequality (1) only.
Any point belonging to region B verifies inequality (2) only.
Any point belonging to region D verifies neither inequality.
Any point belonging to region C verifies both inequalities simultaneously and represents the solution set of the system.

1. Determine graphically the solution set of the following systems.

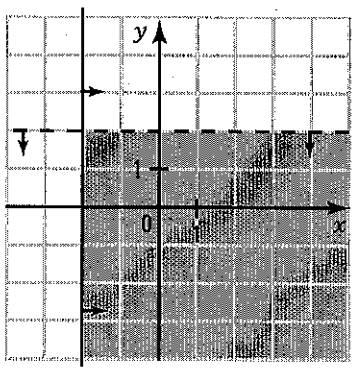
a) $\begin{cases} -4x + y \leqslant -8 \\ x + y \leqslant 2 \end{cases}$



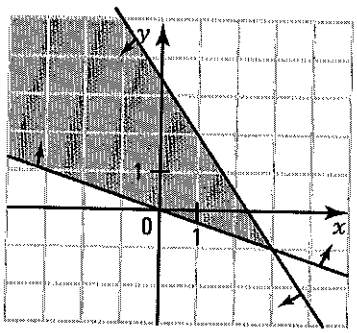
b) $\begin{cases} y > -2x + 3 \\ x \leqslant 2 \end{cases}$



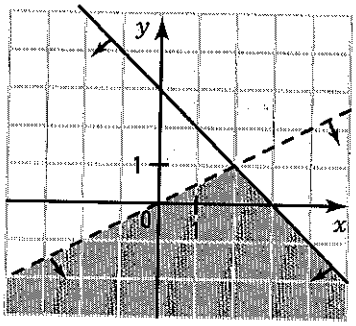
c) $\begin{cases} x \geqslant -2 \\ y < 2 \end{cases}$



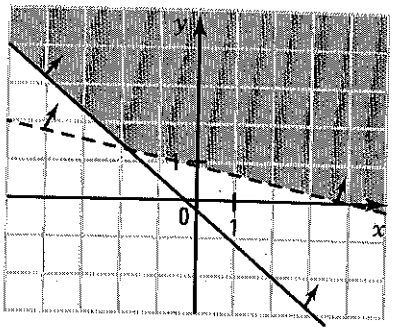
d) $\begin{cases} x + 3y \geqslant 0 \\ 3x + 2y \leqslant 7 \end{cases}$



e) $\begin{cases} x + y \leqslant 3 \\ x > 2y \end{cases}$



f) $\begin{cases} 5x + 6y \leqslant -1 \\ x + 5y > 4 \end{cases}$



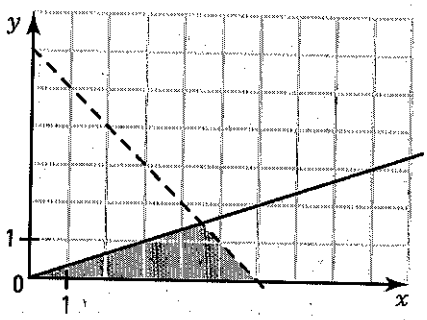
2. In each of the following situations

1. identify the variables involved;
2. write a system that translates the constraints of the situation;
3. represent this system in the Cartesian plane and determine the solution set.

a) A rectangle has a height equal to at least three times its width. Its perimeter is less than 12 cm.

x: length, y: width.

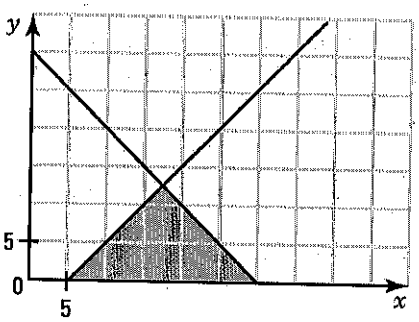
$$\begin{cases} x \geqslant 3y \\ 2x + 2y < 12 \end{cases}$$



b) In an aquarium, there are at least five more fishes as there are plants. The total number of species is at most equal to 30.

x: number of fishes, y: number of plants.

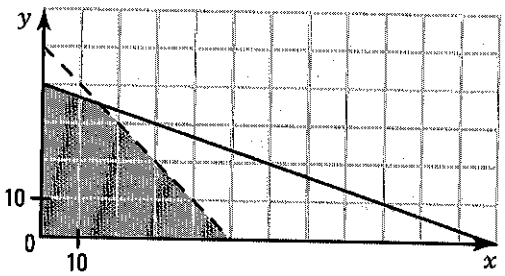
$$\begin{cases} x \geqslant y + 5 \\ x + y \leqslant 30 \end{cases}$$



c) In a 720 m² parking lot, each car occupies an area of 6 m² and each bus an area of 18 m². There are less than 50 vehicles.

x: number of cars, y: number of buses

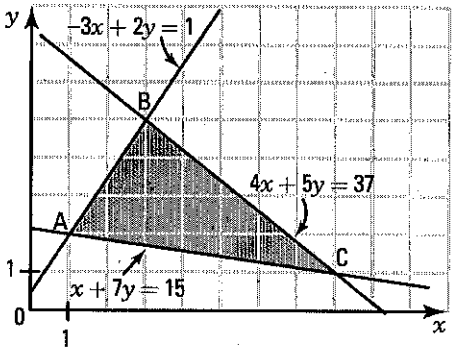
$$\begin{cases} 6x + 18y \leqslant 720 \\ x + y < 50 \end{cases}$$



ACTIVITY 2 Vertices of the polygon of constraints

Consider the following system of inequalities and polygon of constraints.

$$\begin{cases} x \geqslant 0 \\ y \geqslant 0 \\ -3x + 2y \leqslant 1 \\ x + 7y \geqslant 15 \\ 4x + 5y \leqslant 37 \end{cases}$$



a) What system of equations allows you to find vertex A of the polygon of constraints? $\begin{cases} -3x + 2y = 1 \\ x + 7y = 15 \end{cases}$

b) Solve this system using an appropriate method. *A(1, 2)*

c) Determine the polygon's other 2 vertices. *B(3, 5), C(8, 1)*

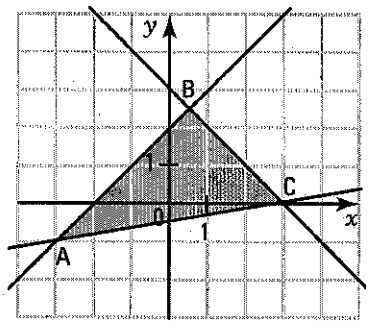
VERTICES OF THE POLYGON OF CONSTRAINTS

To determine the coordinates of the vertices of a polygon of constraints, we solve, for each vertex, the appropriate system of equations.

Ex.: See activity 2

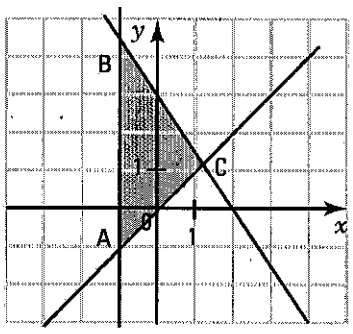
1. Determine the polygon of constraints corresponding to the solution set of each of the following systems of inequalities and find the coordinates of the polygon's vertices.

a) $y \leqslant x + 2$
$x + y \leqslant 3$
$x - 6y \leqslant 3$



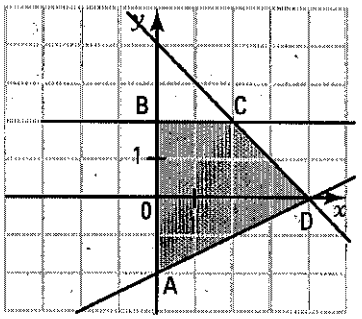
A(−3, −1), B$\left(\frac{1}{2}, \frac{5}{2}\right)$, C(3, 0)

b) $3x + 2y \leqslant 6$
$x \geqslant -1$
$x - y \leqslant 0$



A(−1, 1), B$\left(-1, \frac{9}{2}\right)$ C$\left(\frac{6}{5}, \frac{6}{5}\right)$

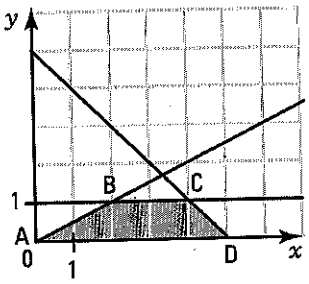
c) $y \leqslant -x + 4$
$x - 2y \leqslant 4$
$x \geqslant 0$
$y \leqslant 2$



A(0, −2), B(0, 2), C(2, 2), D(4, 0)

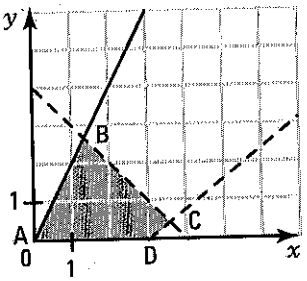
2. Determine the polygon of constraints corresponding to the solution set of each of the following systems of inequalities and find the coordinates of the polygon's vertices.

a) $x \geqslant 0$
$y \geqslant 0$
$x + y \leqslant 5$
$x \geqslant 2y$
$y \leqslant 1$



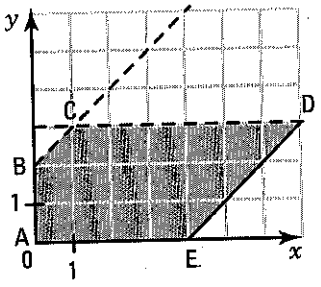
A(0, 0), B(2, 1), C(4, 1), D(5, 0)

b) $x \geqslant 0$
$y \geqslant 0$
$x - y > 3$
$y \leqslant 2x$
$x + y < 4$



A(0, 0), B$\left(\frac{4}{3}, \frac{8}{3}\right)$ C$\left(\frac{7}{2}, \frac{1}{2}\right)$, D(3, 0)

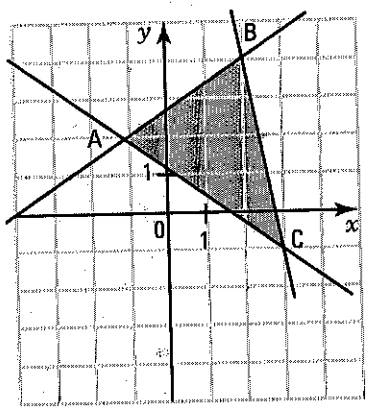
c) $x \geqslant 0$
$y \geqslant 0$
$x - y \leqslant 4$
$y < x + 2$
$y < 3$



A(0, 0), B(0, 2), C(1, 3), D(7, 3), E(4, 0)

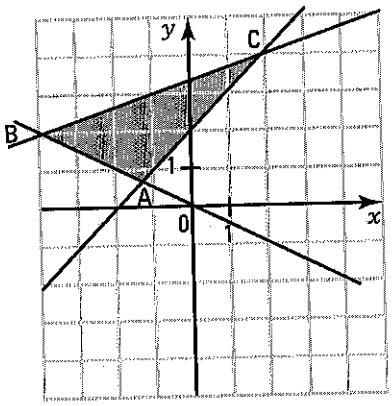
3. In each of the following cases, construct the polygon of constraints corresponding to the system of inequalities and determine, algebraically, the polygon of constraints' vertices.

a) $2x - 3y \geqslant -8$
$5x + y \leqslant 14$
$3x + 4y \geqslant 5$



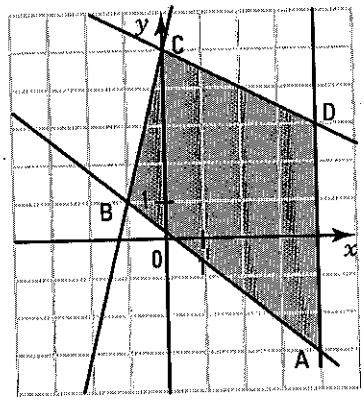
A(–1, 2), B(2, 4), C(3, –1)

b) $y \geqslant x + 2$
$-x \leqslant 2y$
$x - 3y \geqslant -10$



A$\left(-\frac{4}{3}, \frac{2}{3}\right)$, B(–4, 2), C(2, 4)

c) $y \leqslant -\frac{1}{2}x + 5$
$x \leqslant 4$
$4x + 5y \geqslant 1$
$y \leqslant 4x + 5$

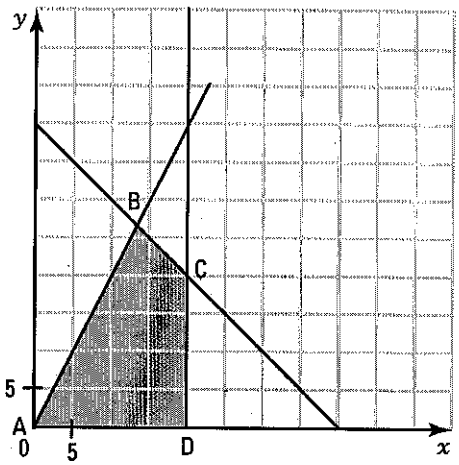


A(4, –3), B(–1, 1), C(0, 5), D(4, 3)

4. For each of the following situations,

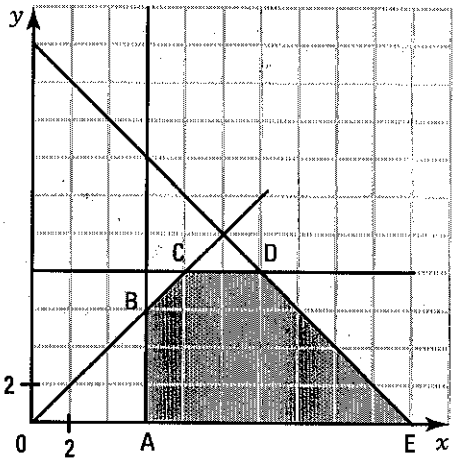
1. identify the variables;
2. determine the system of inequalities that translates the constraints of the situation;
3. construct the polygon of constraints;
4. determine the vertices of the polygon of constraints.

a) A farmer grows tomatoes and potatoes on an area of at most 40 hectares. The area allotted to tomatoes is at most equal to 20 hectares. The area allotted to potatoes is at most equal to twice the area allotted to tomatoes.



***x*: number of hectares allotted to tomatoes.**
***y*: number of hectares allotted to potatoes.**
$x \geqslant 0$
$y \geqslant 0$
$x \leqslant 20$
$x + y \leqslant 40$
$y \leqslant 2x$
$A(0, 0)$, $B\left(\frac{40}{3}, \frac{80}{3}\right)$, $C(20, 20)$, $D(20, 0)$.

b) A sports centre wishes to hire students for its summer camp. To meet the needs of its members, the centre must hire at most 20 students, a minimum of 6 girls, at most as many boys as girls and a maximum of 8 boys.



***x*: number of girls**
***y*: number of boys**
$x \geqslant 0$
$y \geqslant 0$
$x + y \leqslant 20$
$x \geqslant 6$
$y \leqslant x$
$y \leqslant 8$
$A(6, 0)$, $B(6, 6)$, $C(8, 8)$, $D(12, 8)$, $E(20, 0)$.