d) Find the zeros of the following quadratic functions.

1. $f(x) = 2\left(x - \frac{1}{2}\right)^2 - 8$ $-\frac{3}{2}$ **and** $\frac{5}{2}$
2. $f(x) = 2(x - 1)^2 - 10$ $1 - \sqrt{5}$ **and** $1 + \sqrt{5}$
3. $f(x) = -2(x + 3)^2$ **-3**
4. $f(x) = 3(x - 1)^2 + 6$ ***no zeros***

FINDING THE ZEROS – STANDARD FORM

- The number of zeros of the quadratic function $f(x) = a(x - h)^2 + k$ depends on the sign of $-\frac{k}{a}$.
- $-\frac{k}{a} > 0$: There are two zeros x_1 and x_2.

$$\boxed{x_1 = h - \sqrt{-\frac{k}{a}} \text{ and } x_2 = h + \sqrt{-\frac{k}{a}}}$$

- $-\frac{k}{a} = 0$: There is only one zero or the two zeros x_1 and x_2 are equal

$$\boxed{x_1 = x_2 = h}$$

- $-\frac{k}{a} < 0$: There are no zeros.
- Note that the function has no zeros when a and k have the same sign.

Ex.: $f(x) = 2(x - 1)^2 - 8$
$a = 2$; $h = 1$; $k = -8$; $-\frac{k}{a} = 4$
$x_1 = 1 - \sqrt{4} = -1$ and $x_2 = 1 + \sqrt{4} = 3$
The zeros are -1 and 3.

Ex.: $f(x) = 2(x - 1)^2$
$a = 2$; $h = 1$; $k = 0$; $\frac{-k}{a} = 0$
$x_1 = x_2 = 1$
The only zero is 1.

Ex.: $f(x) = 2(x - 1)^2 + 8$
$a = 2$; $h = 1$; $k = 8$; $-\frac{k}{a} = -4$
There is no zero since $-\frac{k}{a} < 0$.

3. Find the zeros of the following functions.

a) $f(x) = -4(x + 2)^2 + 16$ ***-4 and 0***

b) $f(x) = \frac{1}{2}(x + 3)^2 - 2$ ***-5 and -1***

c) $f(x) = 2(x + 1)^2 - 10$ $-1 - \sqrt{5}$ **and** $-1 + \sqrt{5}$ -3.23 1.24

d) $f(x) = (x - 1)^2 - 7$ $1 - \sqrt{7}$ **and** $1 + \sqrt{7}$ -1.65 3.65

e) $f(x) = -2(x + 3)^2$ **-3**

f) $f(x) = 3(x - 2)^2 - 27$ ***-1 and 5***

g) $f(x) = 3(x - 1)^2 + 6$ ***none***

h) $f(x) = -(x + 1)^2$ **-1**

4. Consider the quadratic function $f(x) = a(x - h)^2 + k$.

a) If $a > 0$, indicate the number of zeros when

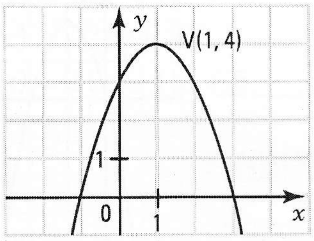
1. $k > 0$. ***none*** 2. $k = 0$. ***only one*** 3. $k < 0$. ***two zeros***

b) If $a < 0$, indicate the number of zeros when

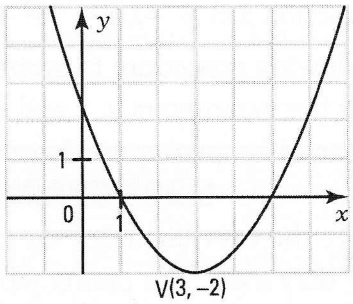
1. $k > 0$. ***two zeros*** 2. $k = 0$. ***only one*** 3. $k < 0$. ***none***

5. Graph the following parabolas.

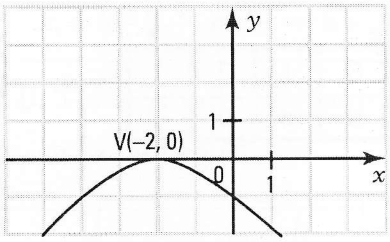
a) $y = -(x - 1)^2 + 4$



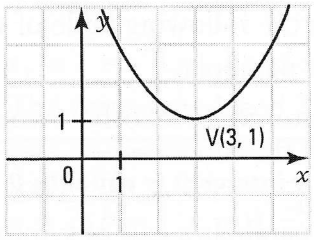
b) $y = \frac{1}{2}(x - 3)^2 - 2$



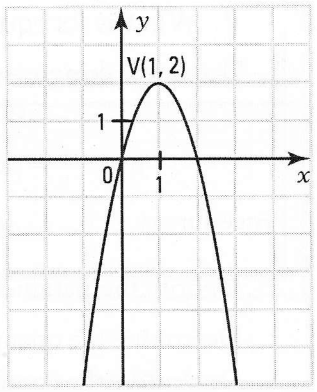
c) $y = -\frac{1}{4}(x + 2)^2$



d) $y = \frac{1}{2}(x - 3)^2 + 1$



6. Consider the function $f(x) = -2(x - 1)^2 + 2$ represented on the right.



a) What is the domain of f? ***dom f = ℝ***

b) What is the range of f? ***ran f =]–∞, 2]***

c) What are the zeros of f? ***0 and 2***

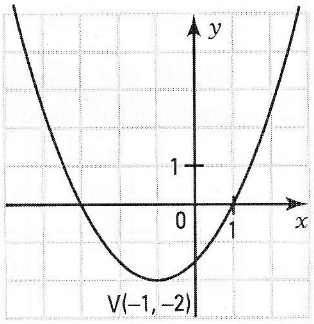
d) What is the y-intercept of f? ***0***

e) What is the sign of f? ***f(x) ≤ 0 if x ∈]–∞, 0] ∪ [2, +∞[***
f(x) ≥ 0 if x ∈ [0, 2]

f) Complete the study of the variation of f.
1. f is increasing over ***]–∞, 1]***. 2. f is decreasing over ***[1, +∞[***.

g) 1. Does function f reach a maximum? If yes, what is it? ***Yes; max f = 2***
2. Does function f reach a minimum? ***no***

7. Consider the function $f(x) = \frac{1}{2}(x + 1)^2 - 2$ represented on the right.



Find

a) 1. dom f. ℝ 2. ran f. ***[–2, +∞[***

b) 1. the zeros of f. ***–3 and 1*** 2. the y-intercept of f. ***–1.5***

c) the sign of f. ***f(x) ≥ 0 over]–∞, –3] ∪ [1, +∞[; f(x) ≤ 0 over [–3, 1]***

d) the variation of f. ***f is increasing over [–1, +∞[.***
f is decreasing over]–∞, –1].

e) the minimum of f. ***min f = –2***

8. Find the domain and range of the following functions.

a) $f(x) = -2(x + 1)^2 + 5$

dom f = ℝ; ran f =]-∞, 5]

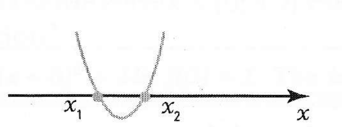
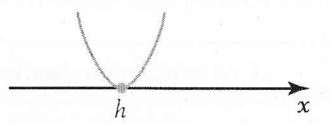
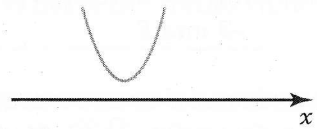
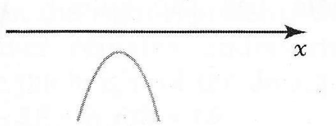
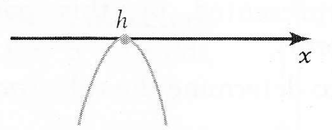
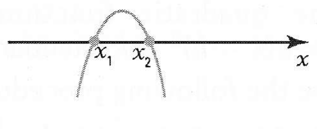
b) $f(x) = \frac{3}{2}(x - 1)^2 - 2$

dom f = ℝ; ran f = [-2, +∞[

9. Determine the zeros of the function $y = -2(x - 3)^2 + 18$. ***0 and 6***

10. What is the y-intercept of the function $y = -2(x - 3)^2 + 7$? ***-11***

11. The sign of the quadratic function $f(x) = a(x - h)^2 + k$ depends on the signs of a and k. Indicate, in each of the 6 following cases, the intervals where $f(x) > 0$ and $f(x) < 0$.

	$k < 0$	$k = 0$	$k > 0$
$a > 0$	 ***f(x) > 0 if x ∈]-∞, x_1 [∪] x_2, +∞[*** ***f(x) < 0 if x ∈]x_1, x_2[***	 ***f(x) > 0 if x ∈ ℝ\{h}***	 ***f(x) > 0, ∀x ∈ ℝ***
$a < 0$	 ***f(x) < 0, ∀x ∈ ℝ***	 ***f(x) < 0 if x ∈ ℝ\{h}***	 ***f(x) > 0 if x ∈]x_1, x_2[*** ***f(x) < 0 if x ∈]-∞, x_1 [∪] x_2, +∞[***

12. Determine, in each case, the values of x for which

1. $f(x) > 0$.
2. $f(x) \geqslant 0$.
3. $f(x) < 0$.
4. $f(x) \leqslant 0$.

a) $f(x) = 2(x - 1)^2 - 2$

1. ***f(x) > 0 if x ∈]-∞, 0 [∪] 2, +∞[***
2. ***f(x) ≥ 0 if x ∈]-∞, 0]∪[2, +∞[***
3. ***f(x) < 0 if x ∈]0, 2[***
4. ***f(x) ≤ 0 if x ∈ [0, 2]***

b) $f(x) = -4(x - 3)^2 + 16$

1. ***f(x) > 0 if x ∈]1, 5[***
2. ***f(x) ≥ 0 if x ∈ [1, 5]***
3. ***f(x) < 0 if x ∈]-∞, 1 [∪] 5, +∞[***
4. ***f(x) ≤ 0 if x ∈]-∞, 1] ∪] [5, +∞[***

13. Determine the values of x for which $y = 3(x - 1)^2 - 27$ is positive.

x ∈]-∞, -2] ∪ [4, +∞[

14. Study the variation of the following functions.

a) $f(x) = 3(x - 1)^2 - 2$

f is decreasing over]-∞, 1].

f is increasing over [1, +∞[.

b) $f(x) = -2(x + 1)^2 + 1$

f is increasing over]-∞, -1].

f is decreasing over [-1, +∞[.

15. Determine the interval over which the function $f(x) = 2(x + 4)^2 + 2$ is increasing. ***[-4, +∞[***

16. Determine the values of x for which the function $y = -3(x + 1)^2 + 12$ is increasing.

x ∈]-∞, -1]

17. In each of the following cases, indicate whether the function reaches a maximum or a minimum and determine it.

a) $f(x) = -2(x - 3)^2 - 1$

A maximum: max f = -1

b) $f(x) = \frac{3}{4}(x + 1)^2 - 2$

A minimum: min f = -2

18. Find the extrema and its nature (maximum or minimum) of $f(x) = 3(x - 1)^2 - 4$.

A minimum; -4

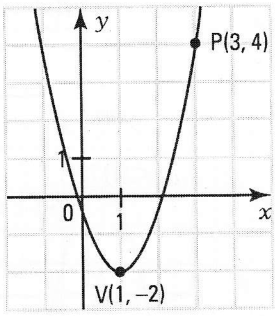
19. What is the axis of symmetry of the parabola with equation $y = (x - 1)^2$?

The line x = 1

20. Find the values of x for which the function $f(x) = -2(x - 1)^2 - 4$ is equal to -36.

-3 and 5

ACTIVITY 10 Finding the rule – Given the vertex and a point



The parabola on the right has the vertex V(1, –2) and passes through the point P(3, 4). The quadratic function represented by this parabola has the rule: $y = a(x - h)^2 + k$ (standard form).

Use the following procedure to determine the rule.

1. Identify h and k. ***h = 1; k = -2***
2. Determine a knowing that the point P(3, 4) verifies the function's rule.

We have: $y = a(x - 1)^2 - 2$

$4 = a(3 - 1)^2 - 2$

$4 = 4a - 2$

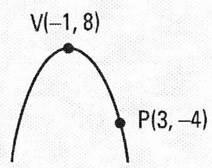
$a = \frac{3}{2}$

3. What is the rule of this function? $y = \frac{3}{2}(x - 1)^2 - 2$

FINDING THE RULE – GIVEN THE VERTEX AND A POINT

$$\boxed{y = a(x - h)^2 + k}$$

1. Identify h and k.	1. $h = -1, k = 8$ $y = a(x + 1)^2 + 8$
2. Find a after replacing x and y in the rule by the coordinates of the given point P.	2. $-4 = a(3 + 1)^2 + 8$ $-4 = 16a + 8$ $a = \frac{-3}{4}$
3. Deduce the rule.	3. $y = \frac{-3}{4}(x + 1)^2 + 8$



21. Determine the equation of the parabola with vertex V and passing through the given point P.

a) V(–1, 4) and P(2, –2) $y = -\frac{2}{3}(x + 1)^2 + 4$

b) V(0, 0) and P(–1, 2) $y = 2x^2$

c) V(2, 0) and P(1, 4) $y = 4(x - 2)^2$

d) V(0, –1) and P(2, 1) $y = \frac{1}{2}x^2 - 1$

22. A parabola with vertex V(3, 16) has a y-intercept equal to 7. What is the y-coordinate of the point A on the parabola whose x-coordinate is 5?

$y = -(x - 3)^2 + 16$; A(5, 12). The y-coordinate of point A is 12.

23. A parabola with vertex V(3, 8) passes through the point A(6, −10). What are the points on this parabola whose y-coordinates are equal to 6?

$y = -2(x - 3)^2 + 8$; $P_1(2, 6)$ and $P_2(4, 6)$

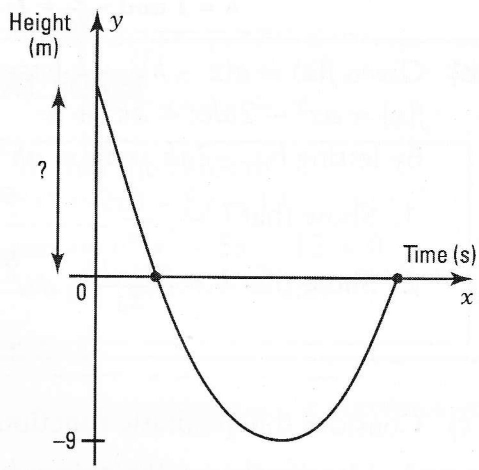
24. What are the zeros of the parabola whose vertex is V(−1, 12) and passes through the point A(2, −15)?

$y = -3(x + 1)^2 + 12$. The zeros are −3 and 1.

25. A parabola with vertex V(6, 10) passes through the point P(10, 6). What is the initial value of this function?

$f(x) = -\frac{1}{4}(x - 6)^2 + 10$; $f(0) = 1$. The initial value is equal to 1.

26. During a competition, a diver enters the water 2 seconds after jumping from the diving board and reaches a maximum depth of 9 m. The portion of the parabola on the right represents the diver's trajectory. If the diver remains underwater for 6 seconds, determine the height of the diving board.



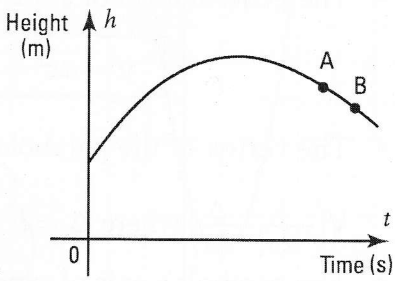
$f(x) = (x - 5)^2 - 9$; $f(0) = 16$

The diving board is at a height of 16 m.

27. At its purchase, a share is worth \$6. We observe that the function f, which gives the value y of the share as a function of the time x in months since its purchase, is a quadratic function. The share reaches a maximum value of \$8 six months after its purchase. What is the value of this share 9 months after its purchase?

$f(x) = \frac{-1}{18}(x - 6)^2 + 8$; $f(9) = 7.5$. The share is worth \$7.50.

28. We have represented on the right the trajectory of two fireworks launched at the same time.
The rule $h = -2(t - 4)^2 + 100$ gives the height h, in metres, as a function of the elapsed time, in seconds, since they were launched. Knowing that firework A explodes at a height of 92 m and that firework B explodes 1 second later, determine at what height firework B explodes at.



Firework A explodes 6 seconds after its launch.

Firework B explodes 7 seconds after its launch at a height of 82 m.