

SIGN OF A QUADRATIC FUNCTION – GRAPHICAL METHOD

To determine the sign of the quadratic function $f(x) = x^2 + x - 6$,

1. we determine the zeros of the function which are then placed on a number line.
2. we draw a sketch of the parabola taking into account its opening which depends on the sign of a .
3. we deduce the sign of the function.
 $f(x) \geq 0$ if $x \in]-\infty, -3] \cup [2, +\infty[$. $f(x) \leq 0$ if $x \in [-3, 2]$.



- 4.** Determine the sign of the following quadratic functions.

a) $f(x) = x^2 + 2x - 15$ $f(x) \geq 0$ if $x \in]-\infty, -5] \cup [3, +\infty[$; $f(x) \leq 0$, if $x \in [-5, 3]$

b) $f(x) = -2x^2 + 7x - 6$ $f(x) \geq 0$ if $x \in [\frac{3}{2}, 2]$; $f(x) \leq 0$ if $x \in]-\infty, \frac{3}{2}] \cup [2, +\infty[$

c) $f(x) = x^2 - 2x + 1$ $f(x) \geq 0$, $\forall x \in \mathbb{R}$

d) $f(x) = -4x^2 + 4x - 1$ $f(x) \leq 0$, $\forall x \in \mathbb{R}$

- 5.** Determine the domain and range of the following functions.

a) $f(x) = -x^2 + 4x + 5$ $\text{Dom } f = \mathbb{R}; \text{ran } f =]-\infty, 9]$

b) $f(x) = x^2 + 2x - 15$ $\text{Dom } f = \mathbb{R}; \text{ran } f = [-16, +\infty[$

- 6.** Study the variation of the following functions.

a) $f(x) = x^2 - x - 6$ $f \nearrow$ if $x \in]-\infty, \frac{1}{2}]$ and $f \nearrow$ if $x \in [\frac{1}{2}, +\infty[$

b) $f(x) = -2x^2 + 3x - 1$ $f \nearrow$ if $x \in]-\infty, \frac{3}{4}]$ and $f \nearrow$ if $x \in [\frac{3}{4}, +\infty[$

- 7.** What are the zeros of the function $y = -3x^2 + 11x - 6$? $\frac{2}{3}$ and 3

- 8.** Find the values of x for which $f(x) = x^2 + 5x - 14$ is positive. $]-\infty, -7] \cup [2, +\infty[$

- 9.** What is the range of the function $f(x) = -x^2 + 2x + 15$? $\text{Ran } f =]-\infty, 16]$

- 10.** What is the y -intercept of $y = 3x^2 - 2x + 5$? 5

- 11.** Find the extrema and its nature (maximum or minimum) for $y = -x^2 - 2x + 3$.
A maximum; 4

- 12.** What is the equation of the axis of symmetry for the parabola $y = -2x^2 + 5x - 3$?

The line with equation $x = \frac{5}{4}$

- 13.** For what values of x is the function $f(x) = 2x^2 - x - 6$ decreasing?

$x \in]-\infty, \frac{1}{4}]$

3.6 Quadratic functions – Factored form

ACTIVITY 1 Quadratic function – Factored form

Consider the quadratic function $f(x) = 2x^2 - 7x + 3$.

- a) What is the value of parameter a ? $a = 2$
- b) Determine the zeros x_1 and x_2 of the function. $x_1 = \frac{1}{2}$ and $x_2 = 3$
- c) The factored form of the quadratic function is $f(x) = a(x - x_1)(x - x_2)$.
Determine the factored form of $f(x) = 2x^2 - 7x + 3$. $f(x) = 2\left(x - \frac{1}{2}\right)(x - 3)$
- d) Expand the factored form to get back to the general form.
 $2\left(x - \frac{1}{2}\right)(x - 3) = (2x - 1)(x - 3) = 2x^2 - 7x + 3$

QUADRATIC FUNCTION – FACTORED FORM

- Given the general form of a quadratic function $f(x) = ax^2 + bx + c$ with zeros x_1 and x_2 .
The factored form of the quadratic function is:

$$f(x) = a(x - x_1)(x - x_2)$$

Ex.: – $f(x) = -2x^2 + 5x - 3$ yields the zeros: $x_1 = \frac{3}{2}$ and $x_2 = 1$.

The factored form of f is $f(x) = -2\left(x - \frac{3}{2}\right)(x - 1)$.

– $f(x) = 4x^2 - 12x + 9$ yields only one zero or two equal zeros: $x_1 = x_2 = \frac{3}{2}$.

The factored form of f is $f(x) = 4\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) = 4\left(x - \frac{3}{2}\right)^2$.

1. In each of the following cases, determine the factored form of the function.

- a) $f(x) = 3x^2 - 5x - 2$ $f(x) = 3\left(x + \frac{1}{3}\right)(x - 2)$
- b) $f(x) = 2x^2 + 7x + 6$ $f(x) = 2\left(x + \frac{3}{2}\right)(x + 2)$
- c) $f(x) = x^2 - 8x + 15$ $f(x) = (x - 3)(x - 5)$
- d) $f(x) = -2x^2 + x + 3$ $f(x) = -2\left(x - \frac{3}{2}\right)(x + 1)$
- e) $f(x) = 4x^2 - 4x + 1$ $f(x) = 4\left(x - \frac{1}{2}\right)^2$