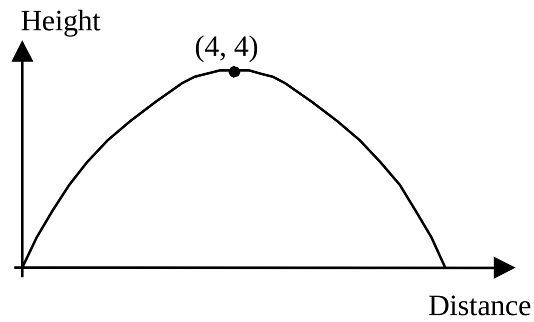
1. The parabolic trajectory (path) of a ball thrown from Pat to Chris is illustrated in the Cartesian diagram below. The maximum height reached by the ball is 4 m.
Which of the following rules of correctly defines this parabola?



A) $y = x^2 - 8x$ C) $y = -0.25\,x^2 - 2x$

B) $y = -4x^2 + 2x$ D) $y = -0.25\,x^2 + 2x$

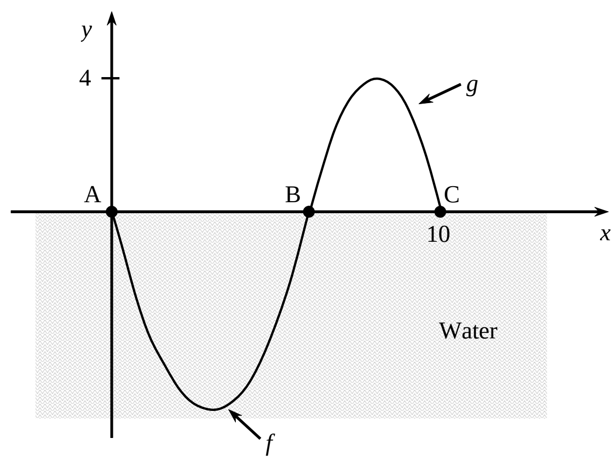
2. What is the equation (rule) of the second-degree function that has a range of $(-\infty, 4]$ and is positive for $x \in\ ]-1, 3[$?

3. What are the zeros of the function $f(x) = x^2 - 2x + 1$?

4. In a Cartesian plane, function f is represented by a parabola. Point P(–7, 172) is one of the points on this parabola, and point V(3, –8) is its vertex. What is the rule of function f?

5. In a Cartesian plane, function f is represented by a parabola. The zeros of function f are 10 and 20, and its minimum is –75. What is the rule of function f?

6. The following graph represents the side view of the path of a dolphin as it performs a trick during a show at an aquarium. This path is composed of portions of two parabolas associated with function f and g respectively. The scale of the graph is in metres. The rule $f(x) = \frac{5}{9}(x-3)^2 - 5$ represents the dolphin's path when it is in the water. When it is out of the water, the dolphin reaches a maximum height of 4 metres. The distance between points A and C is 10 metres. What is the rule of the function g?



7. Determine the equation of the second-degree function associated with the description provided.

a) The vertex is located at $V(3,\ 2)$ and the graph passes through the point $P(4,\ 3)$.

b) The two zeros are -3 and 1 and $f(-1) = 2$.

c) The equation of the axis of symmetry is $x = -1$. The maximum is 2 and the graph passes through the point $P(4,\ -123)$.

d) The only zero of the function is -2 and $f(-1) = -1$.

e) Points $P(-1,\ 7)$, $Q(-9,\ 7)$ and $R(-3,\ 1)$ are on the parabola representing the function.

f) The y-intercept is greater than or equal to the zeros, which are -1 and 5.

1. $V(4,4)$ zeros $(0,0)$, $(8,0)$

$\therefore f(x) = a(x-0)(x-8)$

$f(x) = a(x)(x-8)$

$4 = a(4)(4-8)$

$4 = a(4)(-4)$

$4 = -16a$

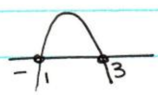
$-0.25 = a$

$f(x) = -0.25x(x-8)$

$= -0.25x^2 + 2x$

$\boxed{\text{Answer : D}}$

2. Ran: $(-\infty, 4]$



$h = \frac{-1+3}{2} = 1$ $\therefore V(1,4)$

$f(x) = a(x-1)^2+4$

$0 = a(3-1)^2+4$

$0 = a(4)+4$

$-4 = 4a$

$-1 = a$

$\boxed{\therefore f(x) = -(x-1)^2+4}$

OR

$f(x) = a(x-x_1)(x-x_2)$

$4 = a(1-(-1))(1-3)$

$4 = a(2)(-2)$

$4 = -4a$

$-1 = a$

$\boxed{f(x) = -(x+1)(x-3)}$

3. $f(x) = x^2-2x+1$ let $y=0$

$0 = x^2-2x+1$

$0 = (x-1)(x-1)$

$x = 1$

$\boxed{\text{The zero is } (1,0)}$

4. $P(-7,172)$ $V(3,-8)$

$f(x) = a(x-h)^2+k$

$172 = a(-7-3)^2-8$

$172 = a(-10)^2-8$

$172 = 100a-8$

$180 = 100a$

$1.8 = a$

$\boxed{f(x) = 1.8(x-3)^2-8}$

5. $x_1 = 10$ $\quad k = -75$ $\quad h = \frac{10+20}{2} = 15$ $\quad \therefore V(15, -75)$

$x_2 = 20$

① $f(x) = a(x-h)^2 + k$

$0 = a(20-15)^2 - 75$

$0 = a(5)^2 - 75$

$0 = 25a - 75$

$75 = 25a$

$3 = a$

$\boxed{f(x) = 3(x-15)^2 - 75}$

② $f(x) = a(x-x_1)(x-x_2)$

$f(x) = a(x-10)(x-20)$

$-75 = a(15-10)(15-20)$

$-75 = a(5)(-5)$

$-75 = -25a$

$3 = a$

$\boxed{f(x) = 3(x-10)(x-20)}$

6. $f(x) = \frac{5}{9}(x-3)^2 - 5$

① find the zeros

$0 = \frac{5}{9}(x-3)^2 - 5$

$5 = \frac{5}{9}(x-3)^2$

$45 = 5(x-3)^2$

$9 = (x-3)^2$

$\pm 3 = x-3$

$3 = x-3$, $\quad -3 = x-3$

$6 = x$ $\quad 0 = x$

② $g(x)$: zeros 6 & 10

max $= 4 = k$

$h = \frac{6+10}{2} = 8$

$g(x) = a(x-8)^2 + 4$

$0 = a(10-8)^2 + 4$

$0 = a(2)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$-1 = a$

$g(x) = a(x-x_1)(x-x_2)$

$4 = a(8-6)(8-10)$

$4 = a(2)(-2)$

$4 = -4a$

$-1 = a$

$\boxed{g(x) = -(x-8)^2 + 4}$

$\boxed{g(x) = -(x-6)(x-10)}$

7. a) $v(3,2)$ $P(4,3)$

$$f(x) = a(x-3)^2 + 2$$
$$3 = a(4-3)^2 + 2$$
$$1 = a(1)^2$$
$$1 = a$$
$$\boxed{\therefore\ f(x) = (x-3)^2 + 2}$$

b) $x_1 = -3$ $P(-1,2)$

$x_2 = 1$

$$f(x) = a(x-x_1)(x-x_2)$$
$$2 = a(-1+3)(-1-1)$$
$$2 = a(2)(-2)$$
$$2 = -4a$$
$$-1/2 = a$$
$$\boxed{\therefore\ f(x) = -1/2(x+3)(x-1)}$$

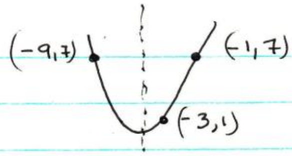
c) $h = -1$ $k = 2$ $P(4,-123)$

$$f(x) = a(x+1)^2 + 2$$
$$-123 = a(4+1)^2 + 2$$
$$-123 = a(5)^2 + 2$$
$$-123 = 25a + 2$$
$$-125 = 25a$$
$$-5 = a$$
$$\boxed{\therefore\ f(x) = -5(x+1)^2 + 2}$$

d) 1 zero = $(-2,0)$ $\Rightarrow$ vertex $v(-2,0)$ $P(-1,-1)$

$$f(x) = a(x+2)^2 + 0$$
$$-1 = a(-1+2)^2$$
$$-1 = a(1)^2$$
$$-1 = a$$
$$\boxed{f(x) = -(x+2)^2}$$

e) P(−1, 7)
Q(−9, 7)
R(−3, 1)



$h = \frac{-9+-1}{2} = \frac{-10}{2} = -5$

$k < 1$

a is (+)

① let $k = 0$

$f(x) = a(x+5)^2$
$1 = a(-3+5)^2$
$1 = a(2)^2$
$1 = 4a$
$1/4 = a$

$f(x) = \frac{1}{4}(x+5)^2$
check $7 = \frac{1}{4}(-1+5)^2$
$7 = \frac{1}{4}(+4)^2$
$7 = \frac{1}{4}(16)$
$7 = 4$ ✗

② let $k = -1$

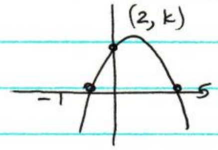
$f(x) = a(x+5)^2 - 1$
$1 = a(2)^2 - 1$
$1 = 4a - 1$
$2 = 4a$
$1/2 = a$

$f(x) = 1/2(x+5)^2 - 1$
$7 = 1/2(-1+5)^2 - 1$
$7 = 1/2(4)^2 - 1$
$7 = 1/2(16) - 1$
$7 = 8 - 1$
$7 = 7$ ✓

works for (−9,7) too.

$\boxed{\therefore\ f(x) = 1/2(x+5)^2 - 1}$

f) zeros : $-1, 5$ $\qquad h = \dfrac{-1+5}{2} = \dfrac{4}{2} = 2$



y-int ≥ 5, so $k > 5$

① $k = 5$ $\qquad f(x) = a(x-2)^2+5$

$0 = a(5-2)^2+5$

$0 = a(3)^2+5$

$-5 = 9a$

$-5/9 = a$ (same answer for $x = -1$)

check $0 = x$

$f(0) = -\frac{5}{9}(0-2)^2+5$

$f(0) = -\frac{5}{9}(4)+5$

$f(0) = -\frac{20}{9}+5 = 2.\overline{7}$ no good

$2.\overline{7} < 5$

② $k = 6$ $\qquad f(x) = a(x-2)^2+6$

$0 = a(3)^2+6$

$-6 = 9a$

$-2/3 = -6/9 = a$

let $x=0$ $\qquad f(0) = -\frac{2}{3}(-2)^2+6$

$= -2/3(4)+6$

$= -8/3+6$

$= 3.\overline{3}$ ×

③ $k = 8$ $\qquad f(x) = a(x-2)^2+8$

$0 = a(3)^2+8$

$-8 = 9a$

$-8/9 = a$

$f(x) = -8/9(x-2)^2+8$ let $x=0$

$f(0) = -8/9(-2)^2+8$

$f(0) = -32/9+8$

$= 4.\overline{4}$ ×

$k = 9$ $\qquad f(x) = a(x-2)^2+9$

$0 = a(3)^2+9$

$-9 = 9a$

$-1 = a$

$f(\ ) = -1(\ -2)^2+9$

$f(0) = -1(-2)^2+9$

$= -4+9$

$= 5$ ✓✓ works

$\therefore \boxed{f(x) = -1(x-2)^2+9}$ *

* other answers are possible

OR f)

zeros: $x_1 = -1$ $x_2 = 5$

$f(x) = a(x+1)(x-5)$

y-int ≥ 5, so let $P(0,5)$ be on the curve.

$5 = a(0+1)(0-5)$

$5 = a(1)(-5)$

$5 = -5a$

$-1 = a$

$\therefore \boxed{f(x) = -1(x+1)(x-5)}$

OR $f(x) = -1(x^2 - 4x - 5)$

$\boxed{f(x) = -x^2 + 4x + 5}$

OR $f(x) = -1(x^2 - 4x) + 5$

$f(x) = -1(x^2 - 4x + 4 - 4) + 5$

$f(x) = -1((x-2)^2 - 4) + 5$

$f(x) = -1(x-2)^2 + 4 + 5$

$\boxed{f(x) = -1(x-2)^2 + 9}$

* other answers are possible (let (0,6) b y-int, etc.)