2. Consider the three forms of a quadratic function: $f(x) = a(x - h)^2 + k$ (standard form). $f(x) = ax^2 + bx + c$, (general form) and $f(x) = a(x - x_1)(x - x_2)$ (factored form).

For each given form, determine the two other forms.

a) $f(x) = 2(x - 1)^2 - 8$

$f(x) = 2x^2 - 4x - 6$ *(general form)*

$f(x) = 2(x + 1)(x - 3)$ *(factored form)*

b) $f(x) = x^2 - 10x + 16$

$f(x) = (x - 5)^2 - 9$ *(standard form)*

$f(x) = (x - 2)(x - 8)$ *(factored form)*

c) $f(x) = 4x^2 - 8x + 3$

$f(x) = 4(x - 1)^2 - 1$ *(standard form)*

$f(x) = 4\left(x - \frac{3}{2}\right)\left(x - \frac{1}{2}\right)$ *(factored form)*

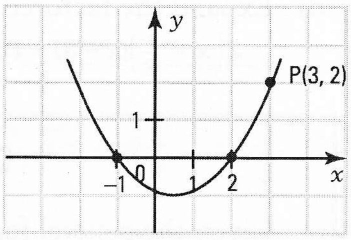
d) $f(x) = 2(x - 1)(x - 5)$

$f(x) = 2x^2 - 12x + 10$ *(general form)*

$f(x) = 2(x - 3)^2 - 8$ *(standard form)*

ACTIVITY 2 Finding the rule – Given the zeros and a point

The parabola on the right has two zeros: –1 and 2 and passes through the point P(3, 2).



The quadratic function represented by this parabola has the rule:
$y = a(x - x_1)(x - x_2)$ (factored form).

Use the following procedure to determine the factored form of the rule.

1. Identify x_1 and x_2. $x_1 = -1$ and $x_2 = 2$
2. Determine a knowing the coordinates of the point (3, 2) verify the rule.

We have: $y = a(x + 1)(x - 2)$

$2 = a(3 + 1)(3 - 2)$

$2 = 4a$

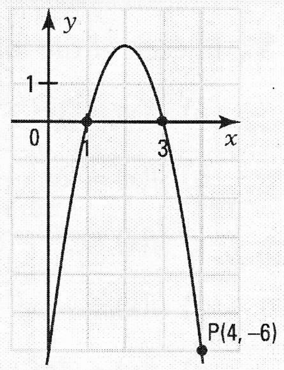
$a = \frac{1}{2}$

3. What is therefore the factored form of the rule? $y = \frac{1}{2}(x + 1)(x - 2)$
4. What is the general form? $y = \frac{1}{2}x^2 - \frac{1}{2}x - 1$

FINDING THE RULE – GIVEN THE ZEROS AND A POINT

$$\boxed{y = a(x - x_1)(x - x_2)}$$

1. Identify the zeros x_1 and x_2.	1. $x_1 = 1$; $x_2 = 3$ $y = a(x - 1)(x - 3)$
2. Determine a after replacing x and y in the rule by the coordinates of the point P.	2. $-6 = a(4 - 1)(4 - 3)$ $-6 = 3a$ $a = -2$
3. Deduce the rule of the function.	3. $y = -2(x - 1)(x - 3)$ (factored form) $y = -2x^2 + 8x - 6$ (general form)



3. Find the rule, in general form, of each of the following quadratic functions.

a) A function with –5 and 2 as zeros and passing through the point P(3, 16).

$y = 2x^2 + 6x - 20$

b) A function with –3 and –1 as zeros and an initial value of –6.

$y = -2x^2 - 8x - 6$

c) A function with the unique zero –2 and passing through the point P(–1, 3).

$y = 3x^2 + 12x + 12$

d) A function with the vertex V(–1, 4) and passing through the point P(2, –5).

$y = -x^2 - 2x + 3$

e) A function with the vertex (1, –8) and one of the zeros equal to 3.

$y = 2x^2 - 4x - 6$

4. What is the vertex of the parabola that has –2 and 4 for zeros and passes through the point A(5, 21)?

$y = 3(x + 2)(x - 4)$; $V(1, -27)$

5. A parabola with zeros –1 and 3 passes through the point A(2, 6). What is the *y*-coordinate of the point B on the parabola that has an *x*-coordinate of 4?

$y = -2(x + 1)(x - 3)$; $B(4, -10)$. *The y-coordinate of point B is –10.*

6. A parabola with zeros –3 and 4 passes through the point A(2, –20). What are the points on this parabola that have a *y*-coordinate equal to 16?

$y = 2(x + 3)(x - 4)$; $P_1(-4, 16)$ and $P_2(5, 16)$

7. What is the *y*-intercept of the parabola with zeros –3 and –1 and passing through the point A(–2, 2)?

The y-intercept is equal to –6.

8. What is the equation of the axis of symmetry of a parabola with zeros –3 and 4?

$x = \frac{1}{2}$

9. The table of values on the right gives the coordinates of different points on a parabola. What is the equation of this parabola?

x	y
0	5
1	8
3	8
5	0

Axis of symmetry: $x = 2$; *The zeros are –1 and 5.*

$y = -(x + 1)(x - 5)$; $y = -x^2 + 4x + 5$

10. Determine the range of the quadratic function *f* with zeros 3 and 5 which verifies $f(2) = -6$.

$f(x) = -2x^2 + 16x - 30$; $V(4, 2)$; $\text{ran } f =]-\infty, 2]$

11. What is the rule of the function *f* that has a range of $]-\infty, 4]$ and is positive over the interval $[-1, 3]$? $f(x) = -x^2 + 2x + 3$

12. The value of a share, in dollars, x weeks after its purchase is given by the rule $y = -0.1x^2 + x + 4.5$. Do you make a profit or a loss if the share is sold two weeks after reaching its maximum value?

Value at purchase: \$4.50; V(5, 7); f(7) = \$6.60. A profit of \$2.10 per share is made.

13. The position $f(t)$, in metres, of a diver relative to the surface is described by the rule $f(t) = 0.5t^2 - 6t + 10$ where t represents elapsed time, in seconds. How long was the diver under water?

$f(t) \leq 0 \Leftrightarrow 2 \leq t \leq 10$. The diver was under water during 8 seconds.

14. The trajectory of a stone thrown from a seaside cliff is a partial parabola. The position $f(t)$, in metres, of the stone relative to sea level is given by $f(t) = -t^2 + 8t + 20$ where t represents elapsed time in seconds since it was thrown. How many seconds after reaching its maximum height will the stone hit the water?

After 6 seconds.

15. The manager of a movie theatre has calculated the following results. When the cost of admission is set at \$10, he observes on average 100 spectators per showing and for each \$0.50 rebate on the admission price, he notices an average of 10 more spectators.

a) Find the rule which gives the total revenue per showing as a function of the number x of \$0.50 rebates.

R(x): Total revenue per showing.

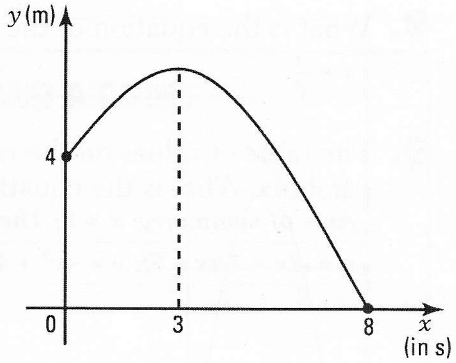
$R(x) = (10 - 0.5x)(100 + 10x)$; $R(x) = -5x^2 + 50x + 1000$

b) 1. At what amount should the manager set the cost of admission in order to maximize the revenue per showing?

The function R reaches its maximum when x = 5. The price of admission should be set at \$7.50.

2. What is the total maximum revenue per showing? ***\$1125***

16. A stone is thrown upward from a height of 4 m. After 3 s, it reaches its maximum height and after 8 s, it hits the ground. Its trajectory is parabolic.



1. What is the maximum height reached by the stone?

6.25 m

2. Determine the elapsed time from the moment the stone was at a height of 2.25 m during its descent to the moment it hit the ground.

1 second