c) The general form of the equation of a circle is: $x^2 + y^2 - 6x + 4y - 12 = 0$.

1. Justify the steps allowing us to write the equation in the standard form.

	Steps	Justifications
1.	$x^2 + y^2 - 6x + 4y - 12 = 0$	- ***General equation of the circle.***
2.	$x^2 - 6x + \ldots + y^2 + 4y + \ldots = 12$	- ***We add 12 to each side.***
3.	$x^2 - 6x + 9 + y^2 + 4y + 4 = 12 + 9 + 4$	- ***We complete in order to obtain perfect square trinomials.***
4.	$(x - 3)^2 + (y + 2)^2 = 25$	- ***We factor the perfect square trinomials to obtain the standard form.***

2. Identify the centre and the radius of the circle. ***Centre: (3, –2); radius: 5.***

EQUATION OF A CIRCLE – GENERAL FORM

- Expanding the standard form of the equation of a circle $(x - h)^2 + (y - k)^2 = r^2$, we obtain the **general form**

$$\boxed{x^2 + y^2 + ax + by + c = 0}$$

Ex.: $(x + 3)^2 + (y - 2)^2 = 25$ (Standard form)
$\Leftrightarrow x^2 + 6x + 9 + y^2 - 4y + 4 = 25$
$\Leftrightarrow x^2 + y^2 + 6x - 4y - 12 = 0$ (General form)

- The curve with equation: $x^2 + y^2 + ax + by + c = 0$ is represented by a circle in the Cartesian plane if and only if $a^2 + b^2 > 4c$.
- From the general form of the equation of a circle, we can find the standard form of the circle equation (See activity 3c)).

10. In each of the following cases, find the equation of circle $\mathscr{C}$ in the standard form and then in the general form.

a) Centre (–1, 3); radius 2.
1. ***$(x + 1)^2 + (y - 3)^2 = 4$***
2. ***$x^2 + y^2 + 2x - 6y + 6 = 0$***

b) Centre (0, –3); radius 2.
1. ***$x^2 + (y + 3)^2 = 4$***
2. ***$x^2 + y^2 + 6y + 5 = 0$***

c) Centre (2, –1); $M(-1, 3) \in \mathscr{C}$.
1. ***$(x - 2)^2 + (y + 1)^2 = 25$***
2. ***$x^2 + y^2 - 4x + 2y - 20 = 0$***

d) $\overline{AB}$ is a diameter, A(–2, 1) and B(4, –3).
1. ***$(x - 1)^2 + (y + 1)^2 = 13$***
2. ***$x^2 + y^2 - 2x + 2y - 11 = 0$***

11. For each of the following circles,
1. write the equation of the circle in the standard form.
2. find the centre and the radius of the circle.

a) $x^2 + y^2 - 2x + 4y - 4 = 0$
$(x - 1)^2 + (y + 2)^2 = 9$
$\omega(1, -2)$; $r = 3$

b) $x^2 + y^2 + 8x + 4y + 19 = 0$
$(x + 4)^2 + (y + 2)^2 = 1$
$\omega(-4, -2)$; $r = 1$

c) $x^2 + y^2 - 4x + 6y - 4 = 0$

$(x - 2)^2 + (y + 3)^2 = 17$

$\omega(2, -3)$; $r = \sqrt{17}$

d) $x^2 + y^2 - x + 3y - 1.5 = 0$

$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 4$

$\omega\left(\frac{1}{2}, \frac{-3}{2}\right)^2$; $r = 2$

12. Explain why each of the following equations is not that of a circle.

a) $x^2 + y^2 - 4x + 6y + 14 = 0$ ***$(x - 2)^2 + (y + 3)^2 = -1$. The sum of 2 squares cannot be negative.***

b) $x^2 - y^2 - 2x - 4y - 19 = 0$ ***The coefficient of y^2 cannot be negative.***

13. Determine, if they exist, the intersection points of circle $\mathscr{C}$ with line l.

a) $\mathscr{C}: (x - 1)^2 + (y + 2)^2 = 9$
$l: y = -x + 2$
(1, 1) and (4, −2)

b) $\mathscr{C}: (x + 1)^2 + (y - 2)^2 = 2$
$l: x - y + 1 = 0$
(0, 1)

c) $\mathscr{C}: (x + 2)^2 + (y + 1)^2 = 1$
$l: x - y - 1 = 0$
No intersection point.

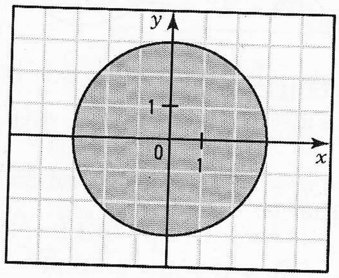
d) $\mathscr{C}: x^2 + y^2 - 2x + 4y + 1 = 0$
$l: x - y - 1 = 0$
(1, 0) and (−1, −2)

e) $\mathscr{C}: x^2 + y^2 = 9$
$l: x + y = 5$
No intersection point.

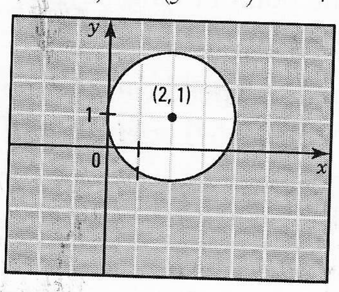
f) $\mathscr{C}: x^2 + y^2 - 2x + 4y - 20 = 0$
$l: 3x + 4y - 20 = 0$
(4, 2)

14. Represent the solution set of the following inequalities in the Cartesian plane.

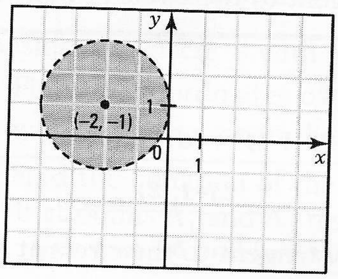
a) $x^2 + y^2 \leqslant 9$



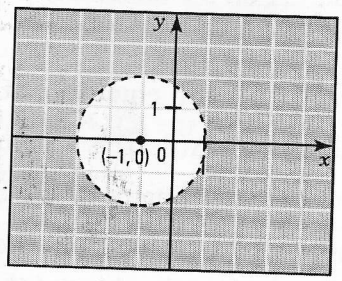
b) $(x - 2)^2 + (y - 1)^2 \geqslant 4$



c) $x^2 + y^2 + 4x + 2y + 1 < 0$

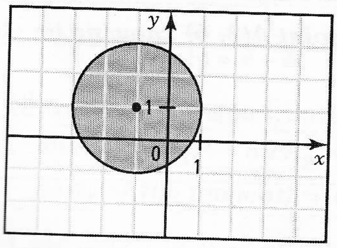


d) $x^2 + y^2 + 2x - 3 > 0$



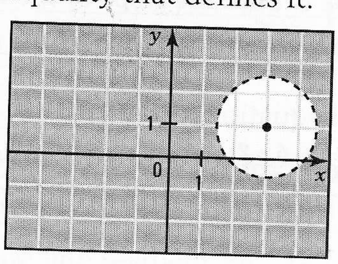
15. For each of the following regions, determine the inequality that defines it.

a)



$(x + 1)^2 + (y - 1)^2 \leqslant 4$

b)



$(x - 3)^2 + (y - 1)^2 > \frac{9}{4}$