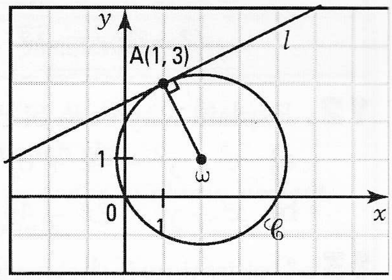
ACTIVITY 4 Line tangent to a circle

Consider on the right the circle $\mathscr{C}$ with equation: $(x-2)^2 + (y-1)^2 = 5$ and point A(1, 3) on this circle.



a) Find the coordinates of the centre ω of this circle. *ω(2, 1)*

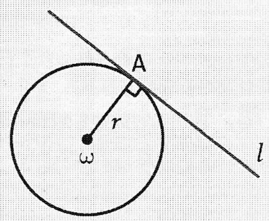
b) Draw the line l passing through point A and perpendicular to the radius ωA.
This line l intersects circle $\mathscr{C}$ in only one point. Line l is called **tangent** to the circle at point A.

c) Find the equation of line l. $y = \frac{1}{2}x + \frac{5}{2}$

LINE TANGENT TO A CIRCLE

- A line is **tangent** to a circle if it intersects the circle in only one point called **point of tangency**.

  Ex.: Given the circle on the right,
  - line l is tangent to the circle at point A.
  - A is the point of tangency.



- **Properties of the tangent:**
  - The tangent is perpendicular to the radius at the point of tangency.

    $$l \perp \overline{\omega A}$$

  - The distance between the centre ω and the tangent l is equal to the radius r.

    $$d(\omega, l) = r$$

  - Given a point A on a circle, there exists **only one line** tangent to the circle at point A.

16. Consider the circle with equation: $(x+1)^2 + (y-2)^2 = 25$.

a) Verify that point A(2, 6) is a point on this circle. $(2+1)^2 + (6-2)^2 = 25$

b) What are the coordinates of the centre ω of the circle? *ω(−1, 2)*

c) Explain the procedure to find the equation of the line l tangent to the circle at point A.

1. We calculate the slope of the radius $\overline{\omega A}$.

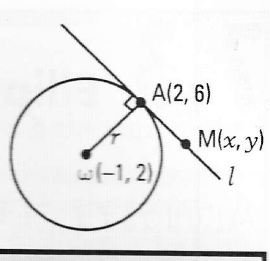
2. We deduce the slope of the tangent l knowing that $\overline{\omega A} \perp l$.

3. We find the equation of the line l passing through point A(2, 6) knowing its slope.

d) Find the equation of the tangent l at point A.

1. $a_{\overline{\omega a}} = \frac{6-2}{2+1} = \frac{4}{3}$ 2. $a_d = \frac{-3}{4}$ 3. $d: y = -\frac{3}{4}x + \frac{15}{2}$

17. Use the data from exercise 16. Justify the steps allowing us to find the equation of the tangent l at point A using the scalar product.



	Steps	Justifications
1.	$M(x, y) \in d \Leftrightarrow \overrightarrow{AM} \perp \overrightarrow{A\omega}$	*Property of the tangent.*
2.	$\Leftrightarrow \overrightarrow{AM} \cdot \overrightarrow{A\omega} = 0$	*Property of the scalar product.*
3.	$\Leftrightarrow (x - 2, y - 6) \cdot (-3, -4) = 0$	*Calculating the vector components.*
4.	$\Leftrightarrow -3(x - 2) - 4(y - 6) = 0$	*Calculating the scalar product.*
5.	$\Leftrightarrow -3x - 4y + 30 = 0$	*Equation of line l (general form).*
6.	$\Leftrightarrow y = -\frac{3}{4}x + \frac{15}{2}$	*Equation of line l (functional form).*

18. Find the equation of the tangent to the circle with equation: $x^2 + y^2 - 2x + 4y - 35 = 0$ at point A(3, 4).

$\mathscr{C}: (x - 1)^2 + (y + 2)^2 = 40$; $\omega(1, -2)$; $a_{\omega A} = 3$; $l: y = -\frac{1}{3}x + 5$

19. Find the equation of the circle with centre $\omega(-2, 3)$ if the circle is tangent to

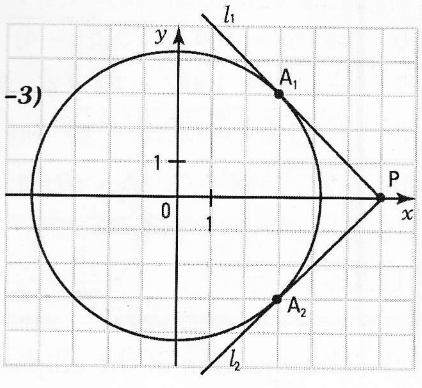
a) the x-axis. ***$(x + 2)^2 + (y - 3)^2 = 9$*** **b)** the y-axis. ***$(x + 2)^2 + (y - 3)^2 = 4$***

20. Consider the line $l: 2x + y - 4 = 0$ and a point $\omega(1, -3)$.

a) Find the equation of the circle $\mathscr{C}$ centred at ω and tangent to line l.
Radius = $d(\omega, l) = \sqrt{5}$; $(x - 1)^2 + (y + 3)^2 = 5$

b) Find the coordinates of the point of tangency A. ***A(3, −2)***

21. Consider the circle $\mathscr{C}$ with equation: $x^2 + y^2 = 18$.



a) Find the coordinates of points A_1 and A_2 on the circle which have an x-coordinate equal to 3. ***$A_1(3, 3)$ and $A_2(3, -3)$***

b) Find the equation of the lines l_1 and l_2 tangent to circle $\mathscr{C}$ at points A_1 and A_2 respectively.
$l_1: y = -x + 6$ and $l_2: y = x - 6$

c) Show that lines l_1 and l_2 meet at a point P located on the x-axis.
The system $\begin{cases} y = -x + 6 \\ y = x - 6 \end{cases}$ has solution P(6, 0)

d) Show that the quadrilateral $0A_1PA_2$ is a square.
$m\overline{0A_1} = m\overline{A_1P} = m\overline{PA_2} = m\overline{0A_2} = \sqrt{18} \Rightarrow 0A_1PA_2$ is a rhombus. $\angle 0A_1P$ is a right angle (Property of the tangent) $\Rightarrow 0A_1PA_2$ is a square, since one of the angles of the rhombus is right.

e) Calculate the area of the region bounded by the line segment PA_1, the line segment PA_2 and the arc of circle A_1A_2. ***Area of square $0A_1PA_2 = 18\ u^2$***
Area of the circular sector $A_10A_2 = \frac{18\pi}{4} u^2$ (one quarter of the disk). Requested area = $\left(18 - \frac{9\pi}{2}\right) u^2$.